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LOYOLA UNIVERSITY CHICAGO

# TOWARDS EVIDENCE-BASED PRACTICES IN MATHEMATICS INSTRUCTION: INVESTIGATING THE IMPACT OF WRITING ON STUDENT ABILITY TO SOLVE MATHEMATICS PROBLEMS

A DISSERTATION SUBMITTED TO

THE FACULTY OF THE GRADUATE SCHOOL

IN CANDIDACY FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

PROGRAM IN SCHOOL PSYCHOLOGY

BY

SHAALEIN C. LOPEZ

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| | |
|---|---|
| ISAT | Illinois Standards Achievement Test |
| ISBE | Illinois State Board of Education |
| NAEP | National Assessment of Educational Progress |
| NCES | National Center for Education Statistics |
| NCLB | No Child Left Behind Act of 2001 |
| NCTM | The National Council of Teachers of Mathematics |

## ABSTRACT

Achievement in mathematics education for students in elementary school through college has lagged behind that of other students internationally. As a result, enhancing mathematics achievement for students in the United States of America has long been a priority. Best practices in teaching across all grades emphasize using instructional methods that have been validated through research, though a review of the literature demonstrates a lack of such substantiated practices in mathematics education. This dissertation attempts to contribute to the field of research-based practices in mathematics education by measuring the effects of an instructional practice long used in mathematics classrooms in grades spanning kindergarten through college: Writing to Learn in the Mathematics Classroom.

A sample of 31 undergraduate students, while engaged in learning three mathematics topics, were assigned to one of three treatment groups to measure the impact of two forms of writing on math learning: The Expository Writing Group; The Novel Problem Writing Group; or the No Writing Group. The purpose of this study was to determine if students learning the same mathematics problem solving methods gained a better understanding of the concepts if the instruction was coupled with one form of writing or another.

Though differences in student posttest means scores between the three writing groups were noted, the differences were not consistent throughout each type of math

problem learned. In addition, no mean differences were found to be significant, even after controlling for differences in prior understanding of the mathematics topics measured by a pretest administered before the instructional period commenced. Despite the lack of significantly different gains in mathematics achievement on the topics under consideration, this study may provide insight into how future studies on the effects of writing on math learning might be designed to better determine if the styles of writing included in this study impact math learning. This study may also, in combination with similar studies with comparable findings, support the notion that the forms of writing included in this study fail to contribute significantly to student gains in mathematics, despite anecdotal support of the practice.

# CHAPTER ONE

# INTRODUCTION

Enhancing the level of mathematics achievement for students in the United States of America has long been a priority expressed not only by education professionals, but by parents of students from kindergarten to college, as well as by various professionals representing a variety of fields from politics to engineering. For decades myriad solutions have been proposed and implemented with variable success, though none appear to have catapulted students in the US from the bottom of international mathematics achievement ranking lists. The path towards effective solutions begins with understanding the fundamental aspects of the problem, and then passes to addressing these aspects thoroughly and systematically. In this chapter I expound upon the problem of poor mathematics achievement among students in the US. I also examine, in a preliminary way, how components of the problem have been addressed in recent years. This discussion will include an introductory illustration of the inadequacy of past efforts to address an essential component of the problem, which will be followed by an examination of the central questions I chose, in consequence, to address through the present research study.

## Poor student achievement in mathematics: A problem of national concern

Recognition of and concern for the relatively poor performance of American students in mathematics is not a recent revelation. Indeed a serious and resounding cry was sounded in the seminal report, “A Nation at Risk”, produced by the National Commission on Excellence in Education (U.S. Department of Education, 1983). In this report it was proclaimed that “the educational foundations of our society are presently being eroded” (¶ 1) due to the mediocre levels of achievement of American students. Poor performance was noted in multiple subject areas including English, Physics, and Mathematics. Specific to mathematics, the report cites a thirteen-year, 40 point decline of average mathematics scores on the Scholastic Aptitude Test (SAT), developed by the College Board and administered to high school students seeking to gain admittance to 4-year colleges or universities (¶ 17). In addition to this disturbing trend, the report cites the finding that only one-third of seventeen-year-old students were found to be able to solve multi-step mathematics problems, an indication that students of this age show little evidence of having mastered higher-ordered thinking as expected (¶ 20). Another disturbing trend described by the US Department of Education in this report is the astonishing 72% increase in the number of remedial mathematics courses offered by four-year-public colleges across the country in the scant five years between 1975 and 1980 (¶ 22). This report provides evidence that we, for decades, have been aware that the academic achievement of students in the United States, in mathematics as well as in other areas, has declined.

Recognizing the need for us to measure student achievement in mathematics and science given the general understanding that high performance in these areas is vital to our nation’s continued ability to innovate, The National Center for Education Statistics (NCES) of the U.S. Department of Education has authorized, for over three decades now, the collection of US student achievement data as part of the National Assessment of Educational Progress (NAEP) project. Since 1990 the NAEP project has published the "Nation's Report Card", which includes a detailed summary of fourth and eighth grade student performance in mathematics. In the year 2003 only 32% of fourth graders and 29% of eighth graders performed at or above the proficient level in mathematics. A student at the proficient level in mathematics is one who can demonstrate mastery of subject-matter knowledge and apply this knowledge to real-world situations. The student performing at the proficient level has also acquired the analytical skills related to the subject matter. In 2005 the NCES reported that the 2003 percentages had risen to 36% of fourth graders and 30% of eighth graders, and by 2007 those proficient or better in mathematics rose to 39% for fourth graders and 32% for eighth graders. Although these percentage increases between the years of 2003 and 2007 are described by the NCES as being statistically significant, it can't be missed that these numbers are indicative of the fact that 61% of fourth graders, and 68% of eighth graders across our nation were not able to demonstrate that they have a proficient level or better of understanding of the mathematics concepts they are expected to learn. This means that the majority of students in these two grades has not gained mastery of grade-appropriate mathematics operations and is furthermore unable to practically use mathematical concepts in real life situations. In addition, it can be seen that the percentage of students performing at or above proficient levels in mathematics has increased by only one to four percentage points every two years (between 2003 and 2007) and that as students advance in age and grade placement their level of mathematics achievement declines; In each year of reporting fewer eighth graders than fourth graders, on average, were measured to be at or above the proficient level in mathematics and this gap widened in each year of reporting.

U.S. Department of education publications related to The No Child Left Behind Act of 2001 (NCLB) also acknowledge and address the poor performance of our students in mathematics. In the NCLB related fact sheet, “The Facts about Math Achievement” (U.S. Department of Education, 2004), Mathematics is described as a “critical skill” that must be improved upon if we are to maintain our nation’s “prosperity” and “security” (¶ 3). The publication continues by noting that though students have made some moves towards improved mathematics achievement in recent years, these improvements are notably modest and precursory (¶ 4). Put simply, the publication asserts that “America’s schools are not producing the math excellence required for global economic leadership and homeland security for the 21st century” (¶ 1). The problem of poor student achievement in mathematics is again proclaimed as a significant problem facing our nation.

The National Mathematics Advisory Panel of the U.S. Department of Education was created by presidential executive order in 2006 to primarily investigate and report upon the elements necessary to prepare students for entry into the study of Algebra (U.S. Department of Education, 2008). In the introductory pages of the Panel’s Final Report, published in 2008, the panel too acknowledges the problem of poor student performance in mathematics across our nation. The report warns that “…without substantial and sustained changes to [our] educational system, the United States will relinquish [our] leadership in the 21st century” (p. *xi*). As part of their review of student performance in mathematics, the Panel found that there appears to be a “falloff” in student achievement in mathematics in late middle school as students either embark upon or otherwise avoid or are excluded from enrollment in Algebra (p. *xiii*). The report further asserts that a “strong grounding” in mathematics at least through Algebra II, if not higher, is needed if individuals hope to gain entry to and complete college, as well as for entree into more lucrative job markets (p. *xii*). Again, poor performance by U.S. students in mathematics is recognized as a portentous national concern.

**Addressing the problem: A View of the field**

As should be expected, we in the United States have moved beyond the mere recognition of the problem of poor student mathematics achievement into the realm of understanding the problem. We have also pushed into the related domain of attempting to make positive change. In an ideal situation, we would come to a complete understanding of the elements needed to affect a positive outcome through educational research, and then we would have judiciously worked to implement the discovered effective elements. In reality however, a consideration of literature in the field reveals a variety of starts and stops, some accompanied by supported by research findings, others not. In other words, our attempts to address the problem of poor student achievement in mathematics appear to have been innocently disorganized at best and recklessly wasteful at worst.

Whatever the results have been, however, we have heard notable cries from various camps of what is needed in order to address the problem. In “A Nation at Risk” (U.S. Department of Education, 1983), in addition to sounding the alarm and predicting the serious implications of poor student achievement, the report acknowledges the significant, though inceptive, efforts to increase support for teachers of science and mathematics (¶ 38). The report also points to, among other things, the need for us to gain an improved understanding of teaching and learning as well as an improved understanding of practices that have worked (¶ 51). Indeed, in the Recommendations section of A Nation at Risk, the commission points to a need for research on teaching, learning and curricula improvement, and highlights the critical role the Federal Government must play in encouraging and supporting these initiatives (¶ 46). Also along this vein of research supporting improved practices, the commission moreover recommends that textbook publishers be required to provide evidence that their materials are effective through collected and statistically analyzed student achievement data (¶ 21). It seems clear that improved teaching practices established through research as well as teacher training to ensure that teachers know how to use these practices has been highlighted as an essential road to improved student achievement more than 25 years ago (and likely more than this if I had chosen to extend my literature search further into the past).

The language of the No Child Left Behind Act of 2001 adds to the cry of what is needed to address the problem of poor student achievement. The NCLB act, in undisputedly clear terms, requires that schools use scientifically-based instructional and assessment methods with long-term evidence of success. Indeed NCLB requires that federal funding go only to those programs whose effectiveness can be supported empirically (U.S. Department of Education, 2001). In its document, “Four Pillars of NCLB” (U.S. Department of Education, 2004), “proven education methods” is highlighted as one of the essential elements of the NCLB act. Within the NCLB act itself, the terms “scientifically-based research”, “scientifically-based instructional methods”, “research-based instructional practices”, or variations of these permeate the document. The use by educators of scientifically-supported practices is paramount. Every element that is supported by NCLB including teacher professional development, extra-curricular student activities, training programs for parents, or intervention programs for students, to name a few, are to be selected and implemented as a result of their demonstrated effectiveness established through research (U.S. Department of Education, 2001). NCLB amplifies the cry made in A Nation at Risk that research is an essential path that leads to improved student achievement.

In the final report of The National Mathematics Advisory Panel (U.S. Department of Education, 2008), the Panel identified six elements that are essential to high achievement of students in mathematics. In a phrase, the Panel purports that “instructional practice should be informed by high-quality research” (U.S. Department of Education, 2008, p. *xiv*). This essential element, combined with another that underscores our nation’s need to create opportunities for the increased undertaking of meaningful studies in educational research in order that educational policy and practice are more optimally executed, point to the recognition by the Panel that more research in education, including mathematics education, is considered requisite. More specifically, the Panel states that “it is essential to produce methodologically rigorous scientific research in crucial areas of national need, such as the teaching and learning of mathematics” (p. *xxvi*). The Panel further establishes the need for increased “large-scale randomized trials” as well as “smaller-scale experiments” in order to further pinpoint instructional materials and designs that work.

The cry for increased mathematics educational research has also come from camps of various sizes that are outside of the purview of the Federal Government. The National Council of Teachers of Mathematics (NCTM) has supported the need for educational research in mathematics in order to, among other things, identify with clarity what mathematics content students are capable of learning based on their developmental level and what pedagogical conditions are needed for this instruction (as cited in Carnine, 2000). In addition and more encompassing, the NCTM’s standing committee on research, the NCTM Research Committee, mandated to describe how educational research can inform improved student achievement in mathematics, published “An Agenda for Research Action in Mathematics Education” (NCTM, 2004). This publication is a call to the community of mathematics educators and educational researchers to put deliberate effort into consolidating what had been established to date in the fields of mathematics education research, to identify continued problems that are in need of investigation, and address how these problems might be attended to through further research.

Others in the field have internalized and echoed the calls made by the NCTM and others. Dougherty (2004), for example, elaborates on the need for increased mathematics education research that addresses the “everyday needs” of mathematics educators (p.75), and further states that research “can tell us under what conditions particular innovations are successful…and what kinds of tasks and questions stimulate learning…” (p. 78).

Carnine (2002) suggests that more research is needed linking specific teaching practices to desired learning outcomes in mathematics. Hiebert et al (2005) warn that increased efforts to hold teachers accountable for student achievement in mathematics will fail absent research efforts that evince changes in instructional practices that promote increased student learning. It is clear that the establishment of effective educational practices in mathematics established through research is a key element in the solution to the problem of poor mathematics achievement among students in the United States.

Through describing the need to increase the quantity and quality of research in mathematics education, those invested have either directly or indirectly alluded to the notion that though some research in the field exists, the current body is insufficient for meeting our nation's need to implement instructional practices that will promote improved understanding among our students. Dougherty (2004), for example concludes that the body of research that exists in the field of mathematics education does not supply evidence that is "compelling enough" to drive comprehensive instructional practice in ways that are needed (p. 75). Latterell et al (2003) point to a decrease in the quantity of published research in the area of mathematics problem solving since the 1980's, while the U.S. Department of Education points to the need to establish research-based teaching methods in mathematics at levels commensurate to those reached in the field of reading education research (2005). As part of its commission, the National Mathematics Advisory Panel reviewed more than 16,000 research publications and policy reports (U.S. Department of Education, 2008). This review, which is described to have covered the past thirty years, produced "surprisingly few methodologically rigorous studies…that examined instructional practices designed to improve the performance of low-achieving students…"(p.49). The Panel also notes that more research is needed to understand how effective teachers move students towards greater gains in learning since existing studies identified by the Panel as "high-quality" fail to sufficiently address these practices (p. *xxi*). It appears clear that the current body of mathematics research is considered deficient for helping move our nation's students towards improved mathematics achievement.

In addition to discussions of the inadequacy of existing research in the field of mathematics education, notable discussion concerning the types of studies needed can be found upon review of the field of literature. Carnine (2000) sheds light onto what is considered a debate in the field over the types of studies that should be developed (i.e. experimental with controls, quasi-experimental, qualitative, etc.). Carnine seems to conclude that descriptive studies are valuable for analyzing problems and building themes while experimental studies are needed to establish generalizable strategies that promote improved student achievement. The National Mathematics Advisory Panel discusses various forms of research though it makes clear that "the primary interest of the Panel is experimental and quasi-experimental research" that considers the effects of instructional interventions (U.S. Department of Education, 2008, p. 84). The Panel designates research that includes random assignment to treatment conditions and control groups as "high-quality" while studies that have non-random samples with control groups are designated to be of "moderate-quality". This expressed preference appears to be in contrast to realities within the field. In a 2005 study by Adler et al analyzed 282 mathematics education research studies published between the years 1999 and 2003 and found that 160 of the studies (57%) could be classified as “small-scale qualitative research”. Of these studies, greater than 60% focused on a single teacher or a group of less than 20 subjects participating in one program or course. Though there appears to be a call for more experimental and quasi experimental studies in mathematics educations, there appears to be a significant lack of such studies in the field currently.

In addition to these types of discussions, others suggest areas within the field of mathematics that should be focused upon by researchers. Danesi (2003) conducted a survey of elementary and high school teachers finding that 83% of respondents indicated that mathematical problem solving was “persistently troublesome” for all students, suggesting a need for improved understanding in this area of mathematics (as cited in Danesi, 2007). Singh et al (2002) point to the need for research designed to measure affective variables for middle-school students given that these variables have been shown to be crucial for student success in high school mathematics. Pape et al (2003) point to the need for additional research in the area of self-regulated learning by students in mathematics, while Desoete et al (2002) point to a need to better understand the essential connection between mathematics and metacognition. The abundance of potential directions for the mathematics education researcher is daunting. The sentiment expressed by Dougherty (2004) that “too often, research in mathematics education does not contribute consciously to an interconnected body or practice-oriented research” (p. 75) seems understandable in light of the breadth of the potential strands of crucial research in mathematics education.

**Writing in mathematics: One road to improved student achievement**

Though the task appears daunting, consideration of the above discussion makes clear a number of salient elements to consider when selecting an appropriate and timely focus for study within the field of mathematics education. It seems imperative that a study designed to inform instructional practice in mathematics is timely. In addition to the above discussion, Carnine (2000) emphasizes that far too often mathematics instructional practices with no research support are implemented. Elbers(2003) draws attention to the need to identify ways for classroom teachers to help students construct their own mathematics understanding. It can also be concluded that more generalizable studies are needed in mathematics education; an element accomplished through the design and implementation of studies that are experimental or quasi-experimental in nature. Even after deciding upon an experimental or quasi-experimental study designed to inform mathematics instructional practices, there remains an uncountable number of potential areas of research. I have identified an instructional practice that has gained in popularity over the past couple of decades despite the incomplete nature of research in the field conducted to justify the practice: writing in the mathematics classroom as a means to support improved student achievement in mathematics.

That writing in mathematics is a research subject of significant import is clear upon consideration of the literature in the field. Seto and Meel (2006) observe that “one of the most significant changes in mathematics pedagogy over the past couple of decades had been the increasing use of writing as a pedagogical tool” (p. 209). The push to use writing to enhance student’s learning of mathematics has swelled to the force of a movement often referred to as the “writing to learn” movement. Steele (2005) explains that the cornerstone of this movement is the notion that when students communicate their thoughts they create mathematical knowledge. Flores and Brittain (2003) put forward that “writing to learn in mathematics means writing to understand, retain, analyze and organize mathematical concepts” (p. 112). These discernments, rightfully so, are often linked to the Vygotskian premise that language and thought become dialectic, each an element in exchanges in reasoning that lead to truth (Vygotsky, 1962). Steele (2005) points out that writing represents an important form of discourse, and that students engaged in writing in the mathematics classroom are “learning to take part in the discourse of mathematics” (p.143). Baxter (2008) echoes this sentiment when he asserts that “writing offers another avenue for students to participate in classroom discourse” (p.37). In its Principals and Standards for school Mathematics, the National Council of Teachers of Mathematics (NCTM) has established communication as an important area of development stating in part that mathematics instruction for all school-aged students “should enable students to organize and consolidate their mathematical thinking through communication, communicate their mathematical thinking coherently…[and] use the language of mathematics to express mathematical ideas precisely (NCTM, 2000, ¶ 1). Specific to writing, the NCTM holds that a student’s ability to write about mathematics should be cultivated in all grades. Writing is also touted as an important form of communication by the NCTM that encourages reflection and moves students towards the formation of clear thought and ideas.

Though expressions in support of the use of writing in mathematics in the classroom abound, numerous utterances of resistance to the practice can also be noted. Though authors have pointed to literature that supports the benefits of writing as a means of learning mathematics (early notable studies include Bell & Bell, 1985; Borasi and Rose, 1989; Boerk; 1990; Contryman, 1992; DiPillo, 1994; and Tsuruda, 1994), limitations of current studies have also been discussed. Notable is Burton & Morgan's (2000) observation that in addition to understanding more about the ways in which writing can be used in the classroom, teachers also need to know which forms of writing are "appropriate for specific purposes" (p.430). In other words, teachers need to know which of the various forms of writing are more or less optimal for particular instructional aims.  Also of note, Ntenza (2006) observed that "a major question that arises is the extent to which writing may assist teachers in determining children's mathematical understanding" (p. 342).  Pugalee (2001), observed that thought there is a push to use writing in the mathematics classroom in order to advance student understanding, "there is insufficient research to provide a basis or rationale for such practices (p. 243). Knowing simply that writing can be beneficial does not represent the type of detailed understanding of the differential effects of writing on mathematics learning that many teachers seek and that many, including the U.S. Department of Education, require. It is timely, then, that researchers choose to design studies that address questions relating to the absolute benefits of the various forms of writing on student learning in mathematics as well as the relative effects on student mathematics achievement when using one form of writing or another. I seek to begin the examination of these principles in the present study. The purpose of this study was to determine if students learning the same mathematical problem solving methods gain a better understanding of the math concepts if the instruction is coupled with a form of writing. My research questions were as follows:

1. Do students required to write expository descriptions of procedures involved in solving problems in mathematics, in addition to learning how to solve these problems, show evidence of better understanding of solving the problems in the short-term than do students who solve the math problems with no writing requirement, or students who solve the problems and engage in another form of writing (novel problem writing), after controlling for differences in initial understanding of the math concepts under consideration?
2. Do students required to write topic-related and task-related novel mathematics problems, in addition to learning how to solve these problems, show evidence of better understanding of solving the problems in the short-term than do students who solve the math problems with no writing requirement, or students who solve the problems and engage in another form of writing (expository writing), after controlling for differences in initial understanding of the math concepts under consideration?

In designing a study that addresses these key questions I acknowledged the call that teachers of mathematics engage in instructional practices that are supported by research findings. I acknowledged and accepted that students in our nation are generally in need of being exposed to teaching methods that are likely to catapult them into the success lane, and that empirically supported teaching methods offer this potential. I recognized that among the myriad instructional solutions that have been supported and attempted, writing in the mathematics classroom to bolster mathematical understanding is considered to have considerable promise. I also acknowledged that despite the existence of writing-to-learn-in-mathematics cheerleaders and preliminary research findings, many teachers express uncertainty about using writing as a pedagogical tool in mathematics (Seto & Meel, 2006). As noted above, research in the field does not adequately address the significance of the effect on student mathematical learning of the various forms of writing often used in the classroom, especially in a comparative way. For these reasons I assert that the purpose of this research study, to consider the relative effects of expository and generative writing in the short term on student mathematics learning, to be highly relevant and decidedly timely.

# CHAPTER TWO

# LITERATURE REVIEW

In moving toward the goal of improved student performance in mathematics, one can embark upon a number of roads. In the estimation of numerous stake holders in the field, the majority of these roads should be paved with materials derived from research. In this chapter I consider the field of literature that addresses writing in the mathematics classroom. I acknowledge that, in general, those contributing to the literature on this topic proffer works that either describe a related research study, illustrate perceived benefits or, to a lesser degree, disadvantages of the practice, or describe in detail ways in which writing has been used in the mathematics classroom. I conclude this chapter by delineating how my study contributes to the body of literature of the field.

## State of the field I: Studies on writing in mathematics

I begin my consideration of studies and other writings in the field that consider the impact of writing as a tool to promote mathematics learning by reiterating the findings of Adler et al (2005) who conducted an analysis of 282 existing studies in mathematics teacher education. The majority of the studies scrutinized were qualitative studies that considered aspects of the instructional practices of single teachers or sample sizes of less than 20 students within single programs. This, combined with the National Mathematics Advisory Panel's finding of surprisingly few methodologically rigorous studies during a consideration of studies of math instructional practices over the last 30 years, forewarns that the number of research studies in the area of writing in mathematics may appear lacking. Indeed I have already noted that there is a need for more supportive evidence established through scientific research in this area; hence my current study.

Though scant for the purposes of driving instruction through research however, studies that consider aspects of using writing in the mathematics classroom exist. A number of studies measuring the perceived benefit of using various forms of writing as a part of mathematics instruction exist. Seto & Meel, for example, in a study conducted in 2006 to measure student perception of the writing assignments they had been given. Forty-six students in 2 college algebra courses were required to use writing throughout the semester. Students were first required to write "mathematical biographies". This form of writing can be placed in the affective category since it required students to chronicle memorable experiences in mathematics and how these experiences made them feel. Students were also required to write "minute papers" throughout the semester whose purposes fit into the expository and affective domains. Students were provided writing prompts to which they responded in kind for a few minutes. As a part of this assignment, students might have been asked to explain a concept such as fractions, how a proffered problem can be solved, or even how they were feeling about a particular current aspect of the course. Students were also required to keep math journals in which they were required to summarize their notes and otherwise reflect on what they had learned. At the end of the semester the 43 students were asked to complete a survey designed to gauge their perceptions of the writing assignments. Thirty-five students completed surveys and results indicated that 74% of responses represented positive feelings in relation to the writing assignments. Survey data also revealed that student preferred writing assignments that fell within the affective domain in comparison to those that probed mathematical understanding.

In another instance, students in a college geometry course were required to write a course textbook over the course of the semester (Boelkins, 2005). This form of writing falls into the generative category of mathematics writing since it required students to write novel problems and solutions. Expository tasks were also a part of this writing requirement since students had to provide textbook-like explanations of geometric concepts. Twenty four students attending a large public university were enrolled in the class under consideration. Though lacking in formality, the instructor (also the author) of this course collected qualitative evidence of student perceptions at the end of the course. The instructor concluded that the majority of students expressed positive feelings in relation to the assignment as evidenced by statements such as “there may have been a lot of work, but the way it was set up actually helped me LEARN the material” (p. 104).

Johanning, a teacher researcher, conducted a study to investigate the effects of writing on students’ mathematical thinking (Johanning, 2000). In this study a group of seven middle school students were asked to share their opinions of the incorporated expository writing components included with their pre-algebra level discussion. Student-expressed sentiment included the notion that writing made students think about meaning in a way they often did not do when just computing.

Another class of qualitative studies evaluates the content of student writing to look for evidence of improved comprehension of the mathematical concepts being taught.

Steele (2005) conducted a qualitative analysis of student learning to understand student algebraic thinking. Eight 7th grade students, including 5 boys and 3 girls, in a small Midwestern city represented Steele's study sample. For the study, the students were instructed to solve 5 linear and 3 quadratic problems over the course of 1 month. The instruction related to each problem lasted 2 to 3 days. As a part of the instruction students were asked to solve the 8 problems and were then required to write about their thinking as it related to the problem. Students also had to explain in writing (i.e. participate in an expository writing task) how they solved or approached the problem. The writing assignment was followed by small-group discussion of student work, including the thoughts and ideas they described in writing. The author of this study concluded that qualitative analysis demonstrated that students had constructed schematic, conceptual, and procedural algebraic knowledge through participation in the writing and discussion activities. The author noted that writing helped to make student algebraic thinking explicit. The author deduced that the study findings support the use of writing in the mathematics classroom.

In another study a qualitative analysis of student writing was undertaken to gain insight into how a group of middle school students think about, solve, or approach math problems (Johanning, 2000). Seven students in a 7th and 8th mixed-grade pre-algebra class for gifted students served as the study's sample. Four of the seven were eighth graders (1 male and 3 female) and three were 7th graders (2 male, 1 female). Over the course of the school year students in the sample responded to writing prompts all related to problem solving seventeen times. The writing was followed by a group discussion and an option for a student to revise or rewrite parts of their original writing. The teacher researcher audio taped and analyzed group discussions in addition to analyzing what the students had written. The writing task could be considered expository since students had to explain how they solved problems through their writing. Results of the qualitative analysis led the researcher to conclude that that the writing component of the problem solving process resulted in meaningful small group discussions that led to “rich learning experiences” (p.151). The researcher further surmises that including writing in mathematics instruction launches students into the act of actively creating and clarifying their thoughts, especially when they know that they will share them later. The writing activity, the author also observed, promoted fluency in the use of the language of mathematics and made students more aware of their mistakes.

Kågesten and Englebrect (2006) conducted a qualitative analysis of the effects of writing on student mathematical understanding. Five beginning engineering students in a technical college mathematics course of 14 chose to participate in the proposed optional writing activities. Students who participated engaged in a written dialogue with the instructors after the instructors initiated the dialogue by commenting upon their solutions given when they took traditional tests. The researchers hypothesized that through writing students would gain a more profound understanding of various calculation procedures because the writing amounted to an opportunity to reflect upon the calculations. The researchers’ analysis led them to conclude that the written dialogue did lead students to understand the concepts more deeply and that the writing assignment led students to see their own mistakes and recognizes gaps in their own knowledge. At the end of the course the researcher also conducted unstructured and semi-structures interviews to, in part, determine if students perceived any benefit to the writing assignments. The researchers reported that almost all students who wrote felt that the writing helped them to understand the mathematics better and liked having the additional time to reflect upon the mathematics. Nine students who did not write were also interviewed in order to gain an understanding why they opted not to participate even though they could have improved their grades.  The researchers found overall that students chose not to write because they did not understand how to do it and because they didn't want to exhibit their poor understanding of mathematics. Results of the study led the researchers to speculate, in conclusion, that students should be exposed to writing in mathematics class on a constant and continual basis starting at early ages and regardless of student ability or achievement levels.

Closely related to studies linking writing in mathematics to improved comprehension, Pegalee (2001) conducted a qualitative analysis of student writing in mathematics as they engaged in problem solved to determine if metacognitive thought was evident in their work. The author worked under the supposition that writing during problems solving is a generative process that necessitates an involvement of "inner speech" (p. 236). Such inner speech, the authors suppose, requires student self-awareness and self regulation, components of metacognition. Pegalee's study was conducted within an introductory high school algebra class, 24 students were enrolled in the class, but data was collected from only 20. Students had been exposed to writing as a part of problems solving 3 months prior to the implementation of the study. During the data collection period students solved one problem a day for six days using writing to describe their problem solving efforts. Pegalee divided the responses into Gaofalo & Lester's (1985) four phases of problem solving, and then looked at each phase for evidence of Gafolo & Lester's metacognitive framework. Pagalee's analysis led him to conclude that "a metacognitive framework was evident in the students writings about their problem solving processes" (p.243). Pagalee used these results as the basis for his conclusion that writing in mathematics is valuable and supports efforts to integrate in into the mathematics classroom because it gives teachers information needed to assess how a student is thinking about the mathematics in which they are engaged.

Pagelee's work has been noted as a logical extension of earlier studies linking writing and metacognition (Steele, 2005). Bell & Bell, for example, in 1985 conducted a study that resulted in the finding that students who wrote were more aware of their thinking, that is, were engaged in metagcognitive thought, and that this engagement led to improved achievement (as cited in Steele, 2005; Stonewater, 2002). Davidson and Steinberg's 1998 finding that writing builds metacognition, which in turn leads to improved problem solving, is another finding linking writing to improved metacognitive thought (as cited in Steele, 2005). Ittigson, (2002) was also an advocate for the notion that writing evokes metacognition.

In another study, Williams (2003) wanted to determine if, after having learned about executive processes related to problem solving, students writing about their uses of executive processes to solve a problem would have an effect on their problem solving ability. In his study, which had a pretest-posttest design, 22 beginning algebra students served as the treatment group; These students were required to write 1-2 paragraphs about how they used executive processes when they solved a particular math problem. Students in the control group (20) just learned about using executive processes to mediate the problem solving process, but were not required to write about their use. Williams found that students in the treatment group made greater gains in terms of their problem-solving abilities than did those in the control group.

In my review of the literature of the field I noticed one study that set out to determine if writing improves conceptual understanding in mathematics and found no difference in the performance of students who engaged in writing and those who did not. Porter and Masingila, (2000) conducted the study under consideration, and included as the study's sample, students enrolled in introductory calculus classes. The treatment group for this study consisted of 3 females and 12 males, while the control group consisted of 4 females and 14 males. Students in the two groups were stated to have comparable levels of readiness for the course. In both instances all students received instruction that took them through a deliberate process of thinking about and discussing various topics in small groups. The control group, however, wrote their thoughts as part of the thinking and discussion process. In essence the writing component addressed the same content as the thinking-discussing component, and the writing appears to have been used as a means to prepare for and reflect about the small group discussions that did not take place in the control group. The researchers administered both the treatment and control groups the same course examinations (4 in total), and classified student errors according to a previously established system of error analysis. Upon classifying the errors, the researchers used error analysis to determine if there was any difference in examination scores among those who used writing as described above and those who did not. The researchers' analysis led them to conclude that there was no difference in the overall performance of students in the treatment and control groups. The authors further suggested that the benefit to using writing as a learning tool in the mathematics classroom appears to be in its generation of student struggles to understand mathematical ideas well enough to explain them to others. The authors suggest that students in the control group were engaged in this struggle to understand and explain to others, though not in writing, and therefore performed as well as students in the treatment group who wrote as a part of their struggle to understand.

**State of the field II: Perceived benefits and formats or writing in the mathematics classroom**

Another class of literature in the field exists whereby educators express, according to their professional experience and expertise, how writing in mathematics is likely to benefit students. Flores & Brittain (2003) and Whitin & Whitin (2002), for example, discuss how students' writings serve as a permanent frozen record of their thoughts that can at any time be revisited and re-reflected upon. These authors also describe writing as a means by which students become active participants in their mathematics learning. Johanning (2000) and Baxter (2008) assert that writing helps students prepare for discussion of mathematics problems, while Ittigson (2002) holds that discussion prior to writing helps students' ideas flow more smoothly when they begin the writing task. Hamdan (2005), Seto & Meel (2006), Baxter (2008) Flores & Brittain (2003), and Albert

& Antos (2000) point to writing in mathematics as a tool of empowerment that builds confidence and gives autonomy and a voice to all students. Baxter (2008), Johanning (2000), Hamdan (2005), and Seto & Meel (2006) discuss how writing contributes to deeper interaction with the mathematics material and improved thinking. Baroody & Bartels (2000), and Baxter (2008) add that writing is a means by which students engage in logical reasoning in mathematics. Whitin & Whitin (2002) point to writing as one way through which students can justify their thinking in mathematics. Aspinwall & Aspinwall (2003), Ediger (2006), Flores & Brittain (2003), Burns (2004) and Seto & Meel (2006) discuss the benefit of using writing in mathematics as a tool for assessment that can serve to give the teacher insight into what students are thinking and how well they understand the material. Albert &Antos (2000), Hamdan (2005), and Seto & Meel suggest that writing in mathematics helps students make a personal connection to math which can lead to a change in student attitude in relation to the subject. Hamdan (2005) suggests that writing in math can have a general therapeutic effect when students are allowed to write about their feelings in relation to the subject. Lee & Herner-Patnode (2007) assert that writing leads to an improved understanding of the language of mathematics, while Baxter (2008), Johanning (2000), and Bratina & Leonard (2003) offer that writing in mathematics leads to improved communications of mathematics ideas and concepts by students. In addition to this encyclopedic range of positions in support of using writing in the mathematics classroom, contrary or skeptical expressions are also noted within the field. Seto & Meel echo an early concern made by Freitay (1997) concerning the time that writing in mathematics takes. Some teachers express concern that writing tasks will take away time from computational practice and problem solving. Ishii (2003) also points to the concern that some teachers have about the level of math understanding required for writing suggesting that writing in math actually requires greater understanding of the material since students need to understand the math being discussed in order to write about it. Seto & Meel (2006) point to expressed concern over whether writing in mathematics actually works to move students towards improved comprehension and how writing can be effectively integrated into classroom instruction. Seto & Meel (2006) also observe that many teachers resist using writing because of an expressed lack of knowledge in relation to how to correctly grade written assignments.

When considering the above discussed studies in the field, I illustrated some ways in which writing has been used as a part of mathematics instruction. An additional group of articles that describe additional ways in which writing has been used in mathematics also populate the literature field. One of the most frequently reported use of writing in mathematics is the keeping of one form or other of a Math Journal. Ishii (2003) and Ediger (2006) describes how journal writing in mathematics is often a means by which students are encouraged to reflect upon an activity or respond to a teacher determined written prompt that is generally aimed at advancing students towards a consolidation of their thinking on some math concept. Seto & Meel (2006) described a similar journal keeping task. In this instance journals were used by students as a tool for reflection upon concepts; students also summarized class notes weekly in their journals. Baxter et al (2002) described their knowledge of journal being used for multiple purposes including to explain thinking, to express feelings or opinions related to mathematics, or to describe or explain math concepts or problem solving approaches (i.e. expository writing). Albert & Antos (2000) describe a daily journal project whereby students recorded the various math problems they encountered as a part of everyday life. The authors assigned this work reportedly as a means of helping students become more comfortable with mathematics in addition to increasing student mathematical understanding. In addition, the authors suggested another purpose that was Vygotskian in nature in that students were being encouraged to become active participants in their learning through journal writing. Hamdan (2005) also described the use of journals as a means to promote active learning among students. Closely related to journal writing, Learning Logs are also described as a means by which students can organize their ideas and work, as well as improve upon their ability to communicate in mathematics (Flores & Brittain, 2003; Bratina & Leonard, 2003). Johanning (2000) used Math Diaries and Cooper (2002) used Mathematical Essays for much the same purposes.

A number of articles describing creative forms of writing in mathematics can also be found. Golembo (2000) had students write creative PEMDAS Stories by which they had to “give life to” the order of operations so that the reader could see the differences in performing operation in different orders in relation to real world events (p. 577). This might mean, for example, illustrating the differences between adding and then raising to a power in comparison to raising to a power then adding. Keller & Davidson (2001) described how Math Poems were used by teachers to encourage students to apply their knowledge of math vocabulary to areas outside of math in order to help students understand the words better and to improve their ability to relate mathematics to real life.

Leitze & Mester (2005) describe how a creative writing assignment motivated by the Kwanzaa holiday was extended to incorporate math concepts such as patterns; the writing was used, the authors assert, as a means by which students had to justify their thinking. Goodman (2005) report how students were required to use personalized written letters to family members to describe the important concepts covered in their beginning calculus class each week. The purpose of this assignment was imparted as helping students to synthesize the concepts covered in the course. Crespo (2003) illustrates how Math Pen Pals were used as a form of writing whereby students created math problems for their peers to solve, explain, and justify. After problems had been solved and explained, the creators of the problems in turn evaluated the work of their peers. Crespo (2004) also supports students writing for audiences other than the classroom teacher suggesting that these activities are “more generative and appealing to students” (p.2). The use of this form of writing in math class was considered useful in that it challenged students to decipher what information was relevant for solving a specified math problem.

In addition to describing instructional episodes of writing in mathematics, a number of authors have also endeavored to summarize the forms of writing that take place in the mathematics classroom. Burns (2004) suggests that writing in math falls into four categories which include keeping journals or logs, solving math problems, explaining mathematical ideas, and writing about learning processes. Johanning (2000) also speaks of expository writing, along with journals, reports, and essays. Ishii (2003) speaks of two forms of writing in mathematics class, journal writing and expository writing. Journal writing is described by Ishii as writing through which students reflect upon an activity or provide a response to a teacher originated prompt. Expository writing, on the other hand, is described as being explanatory or expressive in nature. Baxter (2008) also embarks upon defining expository writing, stating that “expository writing is intended to describe and explain” (p.38). Ntenza (2006) describes “linguistic translation” as another form of writing whereby students translate math symbols and sentences into words (p.355).

Altogether, it is my estimation that discussions on the forms of writing used as a part of mathematics instruction include, in reality, a discussion of the *purposes* for which writing is used, and of the various *modes* of writing encountered. In terms of *purpose,* I speak not of the general purpose of using writing to improve student understanding and performance in mathematics. I speak, instead, of intentions such as illustrating how a math problem was solved, creating a new math problem to be solved by others, explaining feelings related to a math topic, throwing light upon the importance of math concepts in daily life, clarifying a principle of mathematics, and the like. *Modes* of writing include journal, logs, minute papers, essays, biographies, diaries, emails, and a numerous other forms through which some *purpose* is achieved.

**Moving forward: The contribution of my study to the field of literature**

I observe that the majority of research studies in the field on writing and mathematics, as acknowledged before, are qualitative in nature and center around perceived benefits. I point again to the critical notion that more comparative studies that both include control groups, and address the relative effects of writing on mathematics learning are needed in the field. Porter and Masingila (2000) recognized this premise when designing their study (described above). Again, I emphasize that knowing that writing can be beneficial to students does not provide a clear understanding of the differential effects of writing on mathematics learning. This more precise knowledge, I reiterate, is sought by many teachers, and is even required, according to the U.S. Department of Education, when concluding that a particular instructional practice is effective. My study was designed to address this need. I reiterate that the purpose of this study was to determine if students learning the same mathematics problem solving methods gain a better understanding of the math concepts if the instruction is coupled with one form of writing or another. In essence, I generated a series of problem solving lessons coupled with either expository writing to explain how a problem was solved, or novel problem writing to create new problems and solutions. In addition to these treatment groups, I included a control group that used no forms of writing as part of the instruction. This study addressed the problem of poor student performance in mathematics in that it sought to establish (or invalidate) writing in mathematics as a tool that promotes student understanding of mathematical concepts. In addition to contributing to the literature in the field in this way, this study also contributes to the literature by serving as a comparative study that considers the relative effect of using writing or not using writing as part of math instruction, and the relative effects of using one of two forms of writing as part of this instruction.

Knowing that writing as part of mathematics instruction has perceived benefit is an encouraging catalyst towards the initiation of comparative studies on the topic. A review of the literature allows for this knowing. This review also stresses the need for additional studies that can give teachers more direction for using writing in mathematics effectively. I conclude this chapter by reemphasizing my intention to do precisely this through the present study.

# CHAPTER THREE

# METHODOLOGY

I now proceed to a discussion of the methods by which I endeavored to address the problem of poor student achievement in mathematics by means of considering the differential effects of two forms of writing on mathematics learning, outlined previously. In the current chapter I will describe the setting in which my study took place, the participants in the study, and the study instrumentation. I will also detail the procedures used in the implementation of the study.

## Study setting

Loyola University Chicago, a private Jesuit, Catholic University located in Chicago, Illinois, was the setting for the present study. Loyola University is comprised of four campuses and has a student body of more that 15,670 students (2008 data), about 10,124 of whom are undergraduates. Among its various programs, Loyola offers as many as 71 undergraduate majors, 85 master's degrees, and 31 doctoral degrees. In terms of student ethnicity, 29% of Loyola's student population is of African American, Asian American, Latin American, Native American, Multi-racial, or other non-white racial/ethnic backgrounds. The mean grade point average of incoming freshman at Loyola in 2008 was 3.68, while the range of SAT verbal scores for the average incoming freshman was 540 – 648 (for SAT Mathematics the range was 530 – 640). The SOE, in addition to various undergraduate, master's, and doctoral degree programs, offers educational specialist degree programs as well (http://www.luc.edu). In terms of financial data, a reported 92.5% of freshmen who entered Loyola in 2008 received financial assistance; the average assistance award was $19,565 (tuition costs were approximately $29, 486 for entering freshmen in 2008). Students enrolled in programs offered by Loyola University's School of Education (SOE) were participants in the present study. Study components were implemented during class sections held in the Mundelein lecture hall on Loyola University's Lake Shore Campus. The classroom settings (two in total) were small in size with an enrollment of 20 students in each of two sections of a School of Education mathematics course, CIEP 104. Section 1 met on Mondays, Wednesdays, and Fridays from 9:20 a.m. until 10:10 a.m. Section 2 met on Mondays, Wednesdays, and Fridays from 10:25 a.m. until 11:15 a.m. (http://locus.luc.edu).

**Study participants**

As indicated above, participants in this study were freshman undergraduate Loyola University students enrolled in the course CIEP 104. The participants represented a convenience sample since they were selected as potential participants because they were enrolled in two math classes to which I had access. CIEP 104 is a mathematics content course designed for undergraduate students studying to become mathematics teachers. The course is intended to provide students with a foundation for teaching elementary mathematics based on state standards. The course includes a clinical field-based service learning component, and includes among its topics of study, geometry, measurement, data analysis, and probability (http://locus.luc.edu). Of the 40 students enrolled in both sections of CIEP 104, 39 were present when I introduced the study. Of those present, I later noted that all consented to participate in the study and completed the study’s pretest. The student who was absent later consented to participate in the study and took the pretest before classroom instruction related to the study was initiated. Thirty-nine of the forty initial students participants were females, one was male. Nine of the forty initial participants were of minority background (22.5%). Students were also asked to describe previous levels of mathematics taken. Ten students had completed high school mathematics through Algebra II (this included Algebra I, Geometry, and Algebra II), 17 completed high school mathematics through Pre-Calculus, and 12 completed high school mathematics through Calculus. One student completed 4 years of an alternate track of “real-world” mathematics while in high school. In addition to these courses, 7 students took a statistics course, while 1 took a college prep course, and yet another took a college algebra course.

There was some participant attrition by the time the posttest was administered; only 37 students took the study’s posttest. Two participants were absent while one reportedly dropped the course. In addition, not all students completed all of the problem sets that were a part of this study. Six of the 37 students who completed the posttest were eliminated from the study due to failure to complete all three study sets. This elimination was necessary as the problem sets were the participants’ mean to engage in their randomly assigned writing style (or no writing for the control group). Those who did not complete all three problem sets were therefore not considered to have experienced the same level of treatment as those who did. The following table summarizes study element completion by participant. Shaded rows indicate participants who were eliminated:

**Table 1**

**Completion of Study Elements by Participant**

| Study ID | Pretest | Posttest | Lesson 1 | Lesson 2 | Lesson 3 |
|---|---|---|---|---|---|
| 101 | • | • | | | • |
| 102 | • | • | • | • | • |
| 103 | • | • | • | • | • |
| 104 | • | • | • | • | • |
| 105 | • | • | • | • | • |
| 106 | • | • | | | |
| 107 | • | • | • | • | • |
| 108 | • | | • | • | |
| 109 | • | • | • | • | • |
| 110 | • | • | | • | |
| 111 | • | • | • | • | • |
| 112 | • | • | • | • | • |
| 113 | • | • | • | • | • |
| 114 | • | • | • | • | • |
| 115 | • | • | • | • | • |
| 116 | • | • | • | • | • |
| 117 | • | • | • | • | • |
| 118 | • | • | • | • | • |
| 119 | • | • | • | • | • |
| 120 | • | • | • | • | • |
| 121 | • | • | • | • | • |
| 122 | • | • | • | • | |
| 123 | • | • | • | • | • |
| 124 | • | • | • | • | • |
| 125 | • | • | • | • | • |
| 126 | • | • | • | • | |
| 127 | • | • | • | | |
| 128 | • | | | | |
| 129 | • | • | • | • | • |
| 130 | • | • | • | • | • |
| 131 | • | • | • | • | • |
| 132 | • | • | • | • | • |
| 133 | • | • | • | • | • |
| 134 | • | • | • | • | • |
| 135 | • | • | • | • | • |
| 136 | • | • | • | • | • |
| 137 | • | • | • | • | • |
| 138 | • | • | • | • | • |
| 139 | • | • | • | | |
| 140 | • | • | • | • | • |

After Institution Review Board (IRB) approval was gained for the study proposal, I initiated the process of consent with the potential participants. The consent process I followed was governed by Loyola University's recommended consent process phases: Contact; Conversation; and Confirmation (http://www.luc.edu). Executing this essential process for gaining informed subject participation began with approaching students in both sections of CIEP 104 during a class session. Students in these courses were engaged in a conversation as a part of which the purpose, goals, methods, and other relevant information of the study were described in detail. As part of the conversation I asked essential questions whose purpose was to confirm that students considering participation in the study understood the essentials of this study, were capable of giving consent, and were indeed willing to participate in the study. Open-ended questions such as, "Please describe to me your understanding of your role in this study if you choose to participate", and "Please describe for me your options once you agree to participate in the study", are examples of the types of essential questions I asked potential participants. To ensure that the essential elements of the consent process were including (and following university procedures), I developed a Consent Process Script that I followed when engaging both groups of potential participants. This script is included in Appendix A. Once the dialogue of the consent process concluded (about 15 minutes), I asked those willing to participate to sign the study consent form. The consent form can be found in Appendix B. Among other items included on the consent form, students knew that participation in the study was voluntary and that a decline to participate would not impact their course grade in any way. To help assure this, the course instructor was not aware of who was participating in the study, and did not review the student's work or the assigned problem sets in any way. As the researcher, I too was not to be aware of who consented to participate in the study and who did not. Once potential participants were asked to volunteer their participation, I vacated the room while students signed consent forms. The consent forms were placed in a legal-sized envelope before I was prompted by the students to return to the classroom. Students also received an extra copy of the consent form to retain for their records. It happened that all students present in both sections of CIEP 104 consented to participate in the study (as did the one student who was absent), making it evident who was participating in the study. Despite this, I was not aware of the identity of students randomly assigned to the various writing groups of the study, thereby maintaining an aspect of anonymity.

**Study instrumentation**

The majority of mathematics problem solving questions asked of the study participants on the pretest, posttest, and as a part of the lesson problem sets were adapted from a subset of the standard course materials in use by Dr. D. Schiller, the CIEP 104 course instructor; in particular, the textbook and test bank for *Mathematics: A Human Endeavor; Third Edition I* (Jacobs, H.R., 1994). Where problems were not adopted from this text or test bank, they were newly written by the researcher based on a review of items from the Jacobs textbook and test bank. In collaboration with the course instructor, 5 topics were selected as targets for this study. These topics included: Standard Deviation, The Fundamental Counting Principle, Permutations, Combinations, and Determining Probabilities. Participants in the study were first administered a 7-item pretest. Three of the seven pretest and posttest items were divided into two parts, rendering the actual total of responses given to ten. The pretest and posttest were divided into three topic areas covering the five originally selected topics. The following table summarizes the organization of the pretest and posttest:

**Table 2**

**Pretest and Posttest Item Composition**

| Mathematics Topic | Pretest and Posttest Item Numbers |
|---|---|
| Standard deviation | 1a.<br>1b.<br>2.<br>3a.<br>3b. |
| Probability and the Fundamental Counting Principal | 4a.<br>4b. |
| Permutations, Combinations, and Probability | 5.<br>6.<br>7. |

The pretest also included a brief section asking participants to indicate the mathematics courses they had taken in high school by placing a check mark in the appropriate spaces for traditional courses listed (i.e. geometry or algebra II), or by writing in courses not already listed.  With the exception of this section, the pretests and posttests were of identical format, and there was a one-to-one correspondence between the pretest and posttest items in terms of what math skill was needed to solve the particular problem (i.e. question number 4 or the pretest required the same problem solving process as question 4 on the posttest). Items were adapted from existing course textbook problems, and changed only slightly from pretest to posttest, the forms of questions were not changed, but the numbers or situations were altered. For example, on pretest item number 5, trophies were awarded in a film festival, while on posttest item number 5, trophies were awarded in a state fair chili contest. Solutions to each pretest and posttest item were outlined in advance, and verified by a minimum of 4 readers. Students were only asked to find solutions on the pretest and posttest, no additional writing was required. Items were scored so that 1 point was given for correct answers, and zero points for an incorrect answer. The pretest and posttest are included in Appendix C.

In addition to designing and administering a pretest and a posttest, homework problem sets were designed and completed by study participants as the topics were taught by the course instructor. The five topics were divided into three lessons. The following table summarizes the incorporation of the five topics into the lessons of interest to this study.

**Table 3**

**Mathematical Instructional Topics by Study Lesson**

| Study Lesson | Mathematics Topic |
|---|---|
| Lesson 1 | Introduction to Standard Deviation/Determining Standard Deviation |
| Lesson 2 | The Fundamental Counting Principle and Permutations |
| Lesson 3 | Combinations and Determining Probabilities |

In addition to designing homework problem sets, three lessons corresponding to the topics listed in Table 3 were written.  The lessons designed as a part of this study served as a guide for the instructor illustrating the aspects of these topics measured in this study's pretest and posttest, and covered in the study's various problem sets. These lessons can be found in Appendix D. As necessary, the course instructor supplemented instruction to include related topics as necessary to meet the instructional objectives of the course. The comprehension of topics covered in addition to those included in the lessons designed as a part of this study was not measured by this study.

Three homework problem sets were written for each lesson, only one of which was assigned to each student participant based on the study group to which the student had been randomly assigned. In the case that a student chose to revoke their participation in the study, problem sets assigned by the course instructor were also made available. Questions in Problem Set #1 were traditional types of mathematics problems which asked students to find a solution with no additional writing (assigned to participants in the No Writing control group). In addition to finding a solution, exercises in Problem Set #2 asked students to explain in writing how the solution was reached. This type of questioning facilitated an expository engagement with the math topics (assigned to participants in the Expository Writing group). In Problem Set #3 students were asked, in addition to solving problems, to write novel math problems, complete with a solution, which engaged students in a generative form of writing in relation to the mathematics topics (assigned to the Novel Problem Writing group). The fundamental aspect of the mathematics questions in the different problem sets was the same. Students were asked to interface with the material in one of three ways: with no writing, with expository writing, or with novel problem writing. A sample of corresponding questions from the three different problem sets follows. The complete versions of all three problem sets are also included in Appendix D:

*Lesson 3 , Problem Set #1*
*In poker a flush is a hand of five cards, all of the same suit. If there are thirteen spades, how many different flushes consisting of spades are possible?*

*Lesson 3, Problem Set #2*
*In poker a flush is a hand of five cards, all of the same suit. Determine the number of different flushes consisting of spades possible if there are thirteen spades in a deck. Describe in writing how you derived your solution.*

*Lesson 3, Problem Set #3*
*In poker a flush is a hand of five cards, all of the same suit. Write a "word problem" that will require the student to determine the number of different flushes consisting of a specific suit possible if there are thirteen of each suit in a deck. Be sure to give the solution and a rationale for the solution as a part of your response.*

**Figure 1: Example of Corresponding Items in Different Problem Sets**

As illustrated by the above example, participants considered the same concepts in each of the study groups, but interfaced with the concepts in different ways. Concepts were matched from problem set to problem set, though in some instances problems were eliminated in one problem set in order to try to maintain similar estimated completion times. Problems sets were designed to be completed within approximately twenty five minutes. Unlike with the pretest and posttest, items were not scored for correctness, but for completion only. Problem set completion rates were recorded and are included in Table 1.

**Study procedures**

After gaining consent for participation, I administered the pretest. For purposes of the course, the course instructor requested that each student take the pretest. Absent this request, each student in the class would have been given a pretest, but would not have been required to complete it. Each pretest was marked with an identification number that became the student participant's study identification number. A tear-off reminder was provided for each student so that they had a record of this number for use on the posttest and homework problem sets. Upon completion of the pretest, students tore off their study identification numbers and placed their completed pretests in an envelope. Had there been students who declined participation in the study, they would have still received an ID number and would have used this ID when completing their instructor assigned homework. This measure was taken so that both the instructor and I would not be aware of the identities of students participating in the study. Students, when assigned a study ID number, wrote their name and corresponding number on a list provided to the instructor only (Appendix E). When students completed problem sets they used ID numbers (no names) and turned them in to the researcher in an envelope provided. The researcher recorded homework completion rates and reported these rates to the instructor by ID number. The instructor, using the list of study ID numbers by student, recorded homework completion for purposes of course grading. The instructor was unaware of which problem set a student completed, and as a result was unaware of who participated in the study. The researcher knew that all students were participants in the study, but did not know which student completed which problem set.

After the pretest was completed and all participants had received study ID numbers, students were randomly assigned to the three groups of the study (No Writing, Expository Writing, Novel Problem writing). A computerized random number generator was used to categorize study ID numbers into three groups. This process was completing on the same day that the pretest was administered, at which time there were only 39 consenting participants. The 40$^{th}$ participant was randomly assigned to one of the three groups once consent had been obtained and the pretest was completed. The study group assignments by ID number are summarized in the following table:

**Table 4**

**Study Group Random Assignments**

| Group 1<br>Control Group:<br>No Writing | Group 2<br>Expository<br>Writing | Group 4<br>Novel Problem<br>Writing |
|---|---|---|
| 112 | 126 | 140 |
| 130 | 124 | 121 |
| 136 | 104 | 107 |
| 116 | 139 | 113 |
| 132 | 134 | 101 |
| 110 | 105 | 102 |
| 137 | 117 | 118 |
| 109 | 133 | 135 |
| 128 | 120 | 106 |
| 131 | 103 | 111 |
| 119 | 129 | 125 |
| 114 | 127 | 115 |
| 123 | 138 | 108 |
| 122 | | |

Though the study participants represented a convenience sample, by mirroring simple random sampling procedures at the study group assignment level, I hoped to assign the participants to a treatment group in an unbiased way. During the weekend following the pretest administration (the pretest was administered on a Friday), participants received the study group assignments via email; the entire list was emailed to all students enrolled in both sections of CIEP 104 to an email address that they provided on a separate form on the day that the pretest was administered (see Appendix F). Students were also emailed the entire set of homework problem sets to be completed one after each lesson had been covered by the course instructor. Each student was emailed all three versions of the problem sets (corresponding to each group in the study) and instructed to complete only the problem sets for the group to which they had been randomly assigned. Corresponding instructor-assigned problem sets were made available via the electronic blackboard in use by the course instructor for completion by any student who may have chosen to revoke their participation in the study. The first of the three lessons of this study was carried out by the course instructor on the Monday after the administration of the pretest. A summary of the timing of the implementation of the study components is represented in the following table:

**Table 5**

**Study Implementation Timeline**

| Study Time Table | | |
|---|---|---|
| Course Meeting Date | | |
| **Monday** | **Wednesday** | **Friday** |
| **Week 1** | | Consent & Pretest |
| **Week 2**<br>Lesson 1 | | Lesson 2 |
| **Week 3**<br>Lesson 3 | | Posttest |

As the above timeline indicates, the study spanned three calendar weeks and involved 5 course sessions. The corresponding problem sets were turned in when due in an envelope provided. After the three lessons were administered according to the above timeline, I administered the posttest to the study participants. As indicated before, one student was believed to have dropped the course and thus was not present to take the posttest. Two other students were also absent on the day that the posttest was administered. Once the study had been completed, the pretests, posttests and problem sets were made available to the course instructor and students for course purposes.

It is emphasized that all students enrolled in the two sections of CIEP 104 received the same instruction, and that the topics covered during the course of this study were topics that would have otherwise been covered in the course. Students engaged in the writing processes of interest to this study when completing homework problem sets, not in the classroom, and did not receive specific instruction on writing. The nature of the homework problem sets corresponding to each group of the study are summarized below:

**Table 6**

**Nature of Home Work Problem Sets by Study Group**

| Nature of Homework Problem Set | Study Group |
|---|---|
| Solve mathematics problems only | Control Group |
| Solve mathematics problems and explain in writing how the problems were solved | Treatment Group 1: Expository Writing |
| Solve mathematics problems and write new problems with solutions and rationales | Treatment Group 2: Novel Problem Writing |

A final summarization of each phase of the study implementation is provided in the following table:

**Table 7**

**Phases of Study Implementation**

| *Phase* | *Phase Activities* |
|---|---|
| *A. Study introduction & Consent process* | Provide description of the purpose and design of study. Allow for questions. Ask for volunteer participants allowing willing participants to sign informed consent form. |
| *B. Administration of pretest/ Assignment of ID numbers* | Administer pretest (25-30 minutes). Provide ID numbers to study participants to use on pretest. Provide pretest with study ID number to nonparticipants to prevent researcher and instructor from knowing who is participating in the study. |
| *C. Assignment of study groups* | Use random method to assign consenting participants by study ID number to one study group. |
| *D. Implementation of study lessons/Completion of study homework problem sets* | Initiate series of study lessons (instructor). Email homework problem sets (all sets for all groups) in advance of first lesson. Provide instructor-assigned problems for nonparticipants. Instruct students to complete only those study sets designated for their group, and to turn work in by putting it into the provided labeled envelope. Report homework completion rates by ID number to course instructor. |
| *E. Administration of posttest* | Administer posttest (25-30 minutes). Instruct students to include only ID numbers on the posttest. |

**Data analysis**

. The intent of this study was to determine if students, engaged in learning the same mathematical concepts, if required to interface with the material in different ways, gained in understanding of the concepts at different rates. This study's research questions, introduced in chapter one, emphasize my purpose of determining if students engaged in mathematics problem solving in conjunction with either expository writing or novel problem writing achieved solving problems related to standard deviation, the fundamental counting principal, permutations, combinations and related probabilities differently. Subjects were first administered a pretest to gauge initial levels of understanding of these five subtests. Subjects then underwent a period of instruction during which they engaged in expository writing, novel problem writing, or no writing as they solved mathematics problems on these topics. Lastly, subjects were administered a posttest as a measure of achievement in relation to the five topics. To determine if students assigned to the three groups achieved understanding of the material at different rates a one-way univariate analysis of covariance (ANCOVA) was conducted. Data Management and Analysis was performed using PASW Statistics 17.0. The significance level for all analyses was set at the 5% level of significance. During the initial round of data analysis, group mean posttest scores were compared between the three groups using the pretest scores as a covariate; posttest scores served as the dependent variable while group assignment was the fixed factor. Subsequent rounds of data analysis allowed subject-specific posttest scores to be compared after controlling for initial performance as measured by the subject specific pretest questions. The following table summarizes the mean comparisons made using ANCOVA procedures as part of the data analysis process:

**Table 8**

**Executed Mean Comparisons**

| Dependent Variable | Fixed Factor | Covariate |
|---|---|---|
| Total Posttest Score (POSTTOTPRCNT) | Writing Group Assignment (TRMNTGRP) | Total Pretest Score (PRETOTPRCNT) |
| Standard Deviation Posttest Score (POSTSDPRCNT) | Writing Group Assignment (TRMNTGRP) | Standard Deviation Pretest Score (PRESDPRCNT) |
| Fundamental Counting Principal Posttest Score (POSTFCPPRCNT) | Writing Group Assignment (TRMNTGRP) | Fundamental Counting Principal Pretest Score (PREFCPPRCNT) |
| Permutations and Combinations Posttest Score (POSTPCPRCNT) | Writing Group Assignment (TRMNTGRP) | Permutations and Combinations Pretest Score (PREPCPRCNT) |

Prior to conducting the above analyses using ANCOVA procedures, data assumptions related to this procedure were examined to ensure that ANCOVA procedures were appropriate for use with the data set. Assumptions verified were: independence of outcomes; homogeneity of variances; normality; homogeneity of regression slopes; and independence of covariate and treatment effect. The following table summarizes the method by which each assumption was addressed:

**Table 9**

**Methods of ANCOVA Assumptions Verification**

| Assumption | Verification Method |
| --- | --- |
| Independence of Outcomes | Study Design |
| Homogeneity of Variance | Levene's Test of Equality of Error Variances |
| Normality | Shapiro-Wilk Test of Normality of Outcomes (Posttest Scores) |
| Homogeneity of Regression Slopes | Scatterplot: Covariate versus Outcome (Posttest Scores)<br>ANOVA: Custom Model with Fixed Factor (Treatment Group)-Covariate Interaction |
| Independence of Covariate and Treatment Effect | ANOVA: Full Factorial Model with Covariate as Outcome Variable and Treatment Group as Fixed Factor |

The study methodology included in this chapter describes in detail the key aspects of this study including the study setting, the participants in the study, the instrumentation used throughout the study, procedures for implementing the study, and the data analysis procedures used to evaluate the impact of the writing conditions on student achievement in mathematics. In designing and implementing this study as illustrated, I propose that I have taken a step towards evidence-based practices in mathematics instruction; a journey of national significance.

# CHAPTER FOUR

# RESULTS

In this chapter I proceed to the relation of study findings. A description of statistical procedures is followed by the presentation of results. The goal of this analysis was to determine if math problem solving coupled with an expository form of writing, novel problem writing, or no writing resulted in significantly different student performance on a posttest instrument. This consideration was made first on the level of the complete posttest, and then separately on the level of each of the subcategories of test items (i.e. standard deviation, fundamental counting principle, and permutations and combinations). The chapter concludes with a delineation of study findings.

**Statistical tool: ANCOVA**

Univariate one-way analysis of covariance (ANCOVA) is described as a statistical procedure that allows for the comparison of group means on one dependent variable with one fixed factor after a measure of statistical control for one or more variables, the covariate(s), has been applied (Hays, W.L, 1994). As with analysis of variance (ANOVA) procedures, the means of groups subjected to different treatments are compared by calculating sums of squares (SS). More specifically, Total SS is the same as Total Variance, which is in turn equal to the sum of between-group variance (SST or treatment SS) and within-group variance (SSE or error SS). A test statistic ( $F_{observed}$ ) is determined by calculating a version of the ratio of SST to SSE, and is used to determine, taking into account the level of statistical significance selected, the sample size, and the number of treatments, if the difference in the ratio of the sums of squares represents a significant difference in between-group variance, and as an extension, between-group means, despite the differences among means within the group. Unlike ANOVA procedures, ANCOVA procedures take into account the effect of one or more covariates, adjusting the group means to account for this effect, and compares these adjusted means in the manner described. I selected this procedure because it allowed for some statistical mitigation of differences in mathematical understanding of the measured concepts between participants at the start of the study. Theoretically, the removal of this effect allows for a more reliable comparison of between group means.

**Study findings**

Though 37 participants completed the pretest and posttest, 6 were eliminated from data analysis due to not completing all three problem sets prescribed for their group. An individual samples *t*-test was conducted to ensure that there were no differences in the posttest scores of those eliminated and those maintained in the study. A statistical difference in these scores may have had implications for the effects of the writing on student math achievement. The following table illustrates the absence of a significant difference $(\boldsymbol{t = 1.946, p = .06})$. In eliminating the 6 participants, therefore, the analysis of the remaining data was not affected:

**Table 10**

**Independent Samples *t*-test for Equality of Means**

| | t | df | Sig. (2-tailed) |
|---|---|---|---|
| Posttest Total Score as Percent (*Equal variances assumed)* | 1.946 | 35 | .060 |

Note: Test for equality of posttest score means between participants included and eliminated from final data analysis.

The following table (Table 11) represents the number of subjects in each group of the study after subject elimination. Following this table are two additional tables: Table 12, which summarizes posttest means and standard deviations by writing group; and Table 13, which represents adjusted posttest score means resulting after taking into account student pretest scores.

**Table 11**

**Frequency of Subjects per Study Group**

| | Frequency | Percent | Valid Percent | Cumulative Percent |
|---|---|---|---|---|
| No Writing | 11 | 35.5 | 35.5 | 35.5 |
| Expository Writing | 11 | 35.5 | 35.5 | 71.0 |
| Novel Problem Writing | 9 | 29.0 | 29.0 | 100.0 |
| Total | 31 | 100.0 | 100.0 | |

**Table 12**

**Posttest Means and Standard Deviations**

| Writing Group Assignment | | Scores as a Percent | | | |
|---|---|---|---|---|---|
| | | Posttest Total | Posttest Standard Deviation | Posttest Fundamental Counting Principle | Posttest Permutations and Combinations |
| No Writing | Mean | 45.46 | 56.36 | 50.00 | 24.24 |
| | N | 11 | 11 | 11 | 11 |
| | Std. Deviation | 15.08 | 15.02 | 38.73 | 26.21 |
| Expository Writing | Mean | 46.36 | 54.55 | 50.00 | 30.30 |
| | N | 11 | 11 | 11 | 11 |
| | Std. Deviation | 19.633 | 23.82 | 38.73 | 31.462 |
| Novel Problem Writing | Mean | 42.22 | 53.33 | 55.56 | 14.81 |
| | N | 9 | 9 | 9 | 9 |
| | Std. Deviation | 12.0250 | 17.32 | 16.67 | 17.568 |
| Total | Mean | 44.84 | 54.84 | 51.61 | 23.66 |
| | N | 31 | 31 | 31 | 31 |
| | Std. Deviation | 15.678 | 18.60 | 32.87 | 26.096 |

**Table 13**

**Posttest Adjusted Mean Scores**

| Standard Deviation Problems | Fundamental Counting Principal Problems | Permutations and Combinations Problems | Overall Posttest Score |
|---|---|---|---|
| No Writing 56.79 | Novel Problem Writing 54.92 | Expository Writing 29.26 | Expository Writing 46.36 |
| Expository Writing 54.03 | No Writing 50.58 | No Writing 23.99 | No Writing 45.45 |
| Novel Problem Writing 53.45 | Expository Writing 49.9 | Novel Problem Writing 16.40 | Novel Problem Writing 42.22 |

Before conducting the statistical analyses, I considered trends in the posttest data presented in Table 13 above. As is illustrated in this table, there was no consistency in which group performed best on each type of problem, though overall the Expository Writing group performed best. The Novel Problem Writing group performed worst on two out of three of the types of math problems and worst overall, though this group performed best on one type of problem. One possible implication of these trends is that nature of the mathematical problem to be solved could have an impact on how successful a writing-to-learn approach may be. In essence, whether expository writing, novel problem writing, or no writing results in better student understanding may depend on what is being taught.

When beginning the data analysis procedures, I first conducted an analysis to address the following basic question:

*Was there a difference in the rate of learning of three math concepts by 3 different writing groups after controlling for differences in prior understanding of the concepts as measured by a pre-test?*

To determine this I conducted an ANCOVA using pretest scores as the covariate. Before conducting this analysis, however, it was necessary to test the assumptions relevant to this analysis. In this study individual students took the pretest and posttest independently. I assumed then, by virtue of the study design that the assumption of independence had been met. In ANCOVA as in an ANOVA, homogeneity of variance is assumed. To determine if this assumption was met, I considered the results of Levene's Test of Equality of Error Variances, which tests the null hypothesis that the error variance of the variable Posttest Total Score as Percent (POSTOTPRCNT) is virtually equal across the three writing groups. The covariate PRETOTPRCNT (pre test score) as well as the independent variable TRMNTGRP (writing group) were included in this test. The results can be found in the following table (See Appendix G for a description of variables used in the data analysis of this study):

**Table 14**

**Levene's Test for Main ANCOVA**

| F | df1 | df2 | Sig. |
|---|---|---|---|
| .561 | 2 | 28 | .577 |

Note: Tests the null hypothesis that the error variance of the dependent variable is equal across groups. Dependent Variable: Posttest Total Score as Percent

This test does not reveal a significant result ( $\boldsymbol{F_{(2,28)} = .561\, p = .577}$ ); therefore we have met the assumption of homogeneity of variances.

In ANCOVA as in ANOVA, the normal distribution of the dependent variable is assumed. To test for normality of the dependent variable, the Shapiro-Wilk test of Normality was used:

**Table 15**

**Shapiro-Wilk Test of Normality for Main ANCOVA**

| | Statistic | df | Sig. |
|---|---|---|---|
| Posttest Total Score as Percent | .932 | 31 | .049 |

It is observed that the significance value resulting from the Shapiro-Wilk test of Normality is less than .05, a result that is generally considered good evidence that the data set is not normally distributed. ANOVA data analysis procedures are generally accepted as a robust, however, suggesting that though the data fails this test of normality, it can still be relatively reliably analyzed via ANOVA, and therefore ANCOVA procedures.

I now move to a consideration of the assumption of the homogeneity of regression slopes. To begin, I reviewed a scatter plot of the covariate pretest versus the posttest scores to determine visually if the regression slopes appeared to be the same (i.e. if the covariate was the same for all groups). The appropriate scatter plot is as follows:

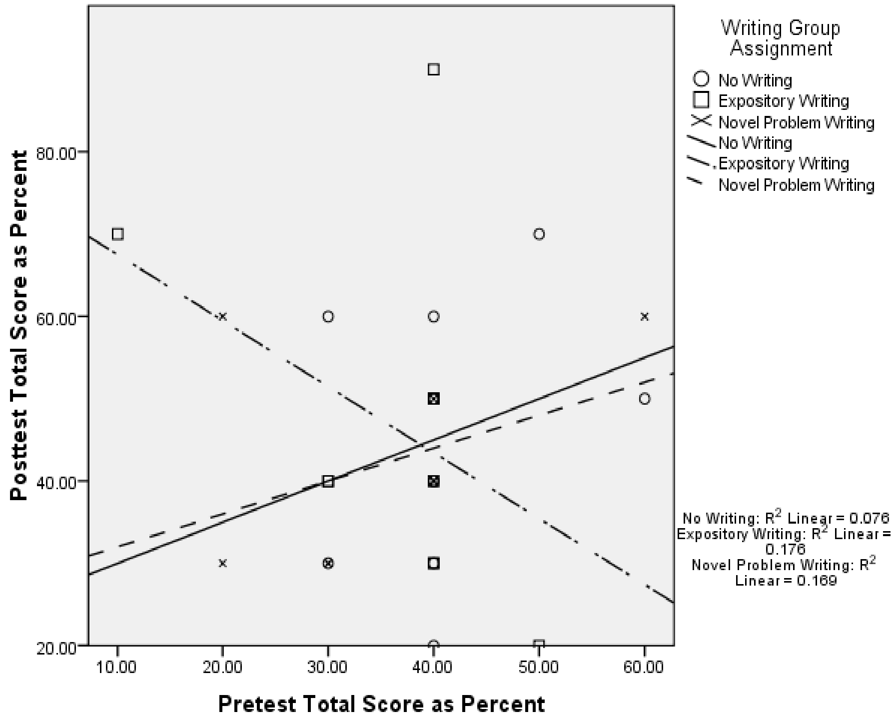



**Figure 2: Covariate Pretest Total Score Versus Posttest Score with Regression lines by Treatment Group**

This figure demonstrates different slopes among the three relationships. Despite this, I can determine if the difference is significant enough to violate the assumption of homogeneity of regression slopes using the appropriate ANOVA procedures. The results are represented in Table 16:

**Table 16**

**ANOVA: Custom Model with Fixed-Factor – Covariate Interaction**

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|
| Corrected Model | 1137.469[a] | 5 | 227.494 | .912 | .489 |
| Intercept | 3301.065 | 1 | 3301.065 | 13.232 | .001 |
| TRMNTGRP | 1103.965 | 2 | 551.982 | 2.213 | .130 |
| PRETOTPRCNT | 3.005 | 1 | 3.005 | .012 | .913 |
| TRMNTGRP * PRETOTPRCNT | 1046.063 | 2 | 523.031 | 2.097 | .144 |
| Error | 6236.724 | 25 | 249.469 | | |
| Total | 69700.000 | 31 | | | |
| Corrected Total | 7374.194 | 30 | | | |

Note: Test of Between-Subjects Effects. Dependent Variable: Posttest Total Score as Percent

I note that the interaction between the writing group and the pretest score is not significant $\boldsymbol{F_{(2,25)} = 2.097,\ p = .144}$. I have therefore met the assumption of homogeneity of regression slopes, despite the visual difference in slopes evident in the above scatterplot.

I conclude consideration of the assumptions of ANCOVA by testing for independence of the covariate and treatment effect. In other words, the covariate should not be different across the treatment groups in the analysis. To do this I run an ANOVA using writing groups (TRMNTGRP) as the independent variable and the covariate, pretest score, as the outcome variable. The results are as follows:

**Table 17**

**ANOVA with Writing Group as Fixed-Factor and Covariate as Outcome**

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|
| Corrected Model | 174.259[a] | 2 | 87.129 | .822 | .450 |
| Intercept | 43458.342 | 1 | 43458.342 | 410.029 | .000 |
| TRMNTGRP | 174.259 | 2 | 87.129 | .822 | .450 |
| Error | 2967.677 | 28 | 105.988 | | |
| Total | 47300.000 | 31 | | | |
| Corrected Total | 3141.935 | 30 | | | |

Note: Test of Between-Subjects Effects. Dependent Variable: Pretest Total Score as Percent.
a. R Squared = .055 (Adjusted R Squared = -.012);

As this table shows, there is no significant effect ( $\boldsymbol{F_{(2,28)} = .822, p = .450}$ ) indicating that the assumption of the independence of the covariate and treatment effect has been met.

Once all ANCOVA assumptions were shown to have been met, I conducted the main analysis to determine whether, after controlling for prior understanding, participation in one of three writing groups resulted in significantly different posttest score means. The results of the ANCOVA used to answer this question are as follows:

**Table 18**

**Main ANCOVA using Full Factorial Model:**

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. | Partial Eta Squared | Observed Power[b] |
|---|---|---|---|---|---|---|---|
| Corrected Model | 91.407[a] | 3 | 30.469 | .113 | .952 | .012 | .068 |
| Intercept | 3945.453 | 1 | 3945.453 | 14.627 | .001 | .351 | .958 |
| PRETOTPRCNT | .042 | 1 | .042 | .000 | .990 | .000 | .050 |
| TRMNTGRP | 90.930 | 2 | 45.465 | .169 | .846 | .012 | .073 |
| Error | 7282.787 | 27 | 269.733 | | | | |
| Total | 69700.000 | 31 | | | | | |
| Corrected Total | 7374.194 | 30 | | | | | |

Note: Test of Between-Subjects Effects. Dependent Variable; Posttest Total Score as Percent. a. R Squared = .012 (Adjusted R Squared = -.097). b. Computed using alpha = .05

This analysis indicated that there was no significant difference in the means of the three writing groups (TRMNTGRP) after accounting for differences in initial understanding of the measured concepts represented by the pretest scores ($\boldsymbol{F_{(2,27)} = .169, p = .846}$). Considering the mean posttest score values, both original and adjusted (see Table 19 below), I noted that there were some differences in these mean values, but the above analysis indicated that the differences were not significant. In noting the value of the power statistic in Table 18 (.073), I observed that this low power value indicated that there is a 92.7% chance that this analysis failed to detect an effect that may be there. In essence, though ANCOVA results indicated that there is no significant differences in the group mean posttest score values, it is possible that elements of this study design, such as sample size, were not optimal for detecting an effect. An increased sample size, for example, could result in improved power, which would allow for a more confident interpretation of significance results.

**Table 19**

**Posttest Means and Adjusted Means by Writing Group**

| Writing Group Assignment | N | Original Mean | Adjusted Mean |
|---|---|---|---|
| No Writing | 11 | 45.4545 | 45.466[a] |
| Expository Writing | 11 | 46.3636 | 46.358[a] |
| Novel Problem Writing | 9 | 42.2222 | 42.214[a] |

Note: Dependent Variable: Posttest Total Score as Percent. a. Covariates appearing in the model are evaluated at the following values: Pretest Total Score as Percent = 37.7419.

Another aspect revealed by this analysis was its effect size. This analysis yielded a partial Eta-squared value of .012 on the factor TRMNTGRP, suggesting that the factor TRMNTGRP only accounted for 1.2% of the variance in group means (adjusted). This small effect size indicates that the writing treatments implemented in this study produced a small effect on student learning as measured by the posttest. In essence, students in the No Writing group, the Expository Writing Group, and the Novel Problem Writing group performed at virtually the same level.

After having found that there was no significant difference in the total posttest scores of the different writing groups, I wished to consider if there was any difference between group-understanding of each type of math problem being taught. The math problems fell into three categories: standard deviation, the fundamental counting principle, and combinations and permutations. I simulated the procedures detailed in the above analysis three separate times to determine if, after controlling for initial differences in baseline understanding of each concept (measured by the pretest), there were differences in performance on the posttest between the three writing groups. In essence, I performed three additional and distinct analyses that addressed the following questions:

> *Was there a difference in the rate of learning of math problems covering concepts related to standard deviation by 3 different writing groups after controlling for differences in prior understanding of the concepts as measured by a pre-test?*
>
> *Was there a difference in the rate of learning of math problems covering concepts related to the fundamental counting principle by 3 different writing groups after controlling for differences in prior understanding of these concepts as measured by a pre-test?*
>
> *Was there a difference in the rate of learning of math problems covering concepts related to permutations and combinations by 3 different writing groups after controlling for differences in prior understanding of the concepts as measured by a pre-test?*

For each analysis I first considered the five ANCOVA assumptions. All assumptions were met in each analysis with the exception of the assumption of normality. In each analysis the data were found to deviate from normal distribution. The analyses were continued, however, given the robust nature of ANCOVA procedures. Results of the various Shapiro-Wilk Test of Normality for each analysis are summarized in the following table:

**Table 20**

**Shapiro-Wilk Test of Normality for Additional Analyses**

| | Statistic | Df | Sig. |
|---|---|---|---|
| Posttest Standard Deviation Score as Percent | .859 | 31 | .001 |
| Posttest Fundamental Counting Principle Score as Percent | .794 | 31 | .000 |
| Posttest Permutations and Combinations Score as Percent | .784 | 31 | .000 |

Each ANCOVA conducted indicated that there was no significant difference between the group mean test scores in each analysis. The following table illustrates the various $F_{observed}$ and significance for each analysis, along with the associated effect size (partial-Eta-squared) and power statistics:

**Table 21**

**$F_{observed}$, Significance, Eta-squared, and Power Values by Analysis**

| | Standard Deviation Posttest Score Analysis | Fundamental Counting Principal Posttest Score Analysis | Permutations and Combinations Posttest Score Analysis |
|---|---|---|---|
| $F_{observed}$ | .084 | .060 | .578 |
| Sig. | .919 | .942 | .568 |
| Eta-squared | .006 | .004 | .041 |
| Power | .062 | .058 | .136 |

In essence, the results in Table 21 indicate that subjects in the three writing groups of the study performed virtually the same on mathematics problems related to standard deviation, the fundamental counting principal, and combinations and permutations. As with the original analysis, the power of each analysis was found to be low, suggesting a high probability that the analyses failed to detect an effect that may have been present. These probability values were 93.8% for the Standard Deviation analysis, 94.2% for the Fundamental Counting Principal analysis, and 86.4% for the Permutations and Combinations analysis. These values indicate that the ANCOVA analysis related to the fundamental counting principal problems had the greatest power. Consideration of Eta-squared values indicate that this analysis also had the largest effect ($\eta^2 = .041$), though this effect is considered small by conventional standards.

Overall then, the ANCOVA procedures executed as a part of this study and outlined in this chapter indicate that, after controlling for initial differences in performance, no significant differences in mean posttest group scores were found. Not only were there no significant differences between total mean posttest group scores, but separate ANCOVA analyses indicated that there were no significant differences in how each writing group performed on each type of math problem (standard deviation, fundamental counting principal, and permutations and combinations). Power statistics related to the analyses performed suggest that the current study lacks sufficient power to detect any significant effects, even if they actually exist. In addition, measures of effect size indicate that subjects in the various groups performed virtually the same; the largest effect size was noted in the analyses of performance on the permutations and combinations math problems, but even this effect is considered small ($\eta^2 = .041$)..

# CHAPTER FIVE

# DISCUSSION

The purpose of this study was to determine if two forms of writing, novel problem writing or expository writing, in comparison to solving problems in the traditional manner by requiring no such writing, had an effect on student learning of various mathematical concepts (standard deviation, the fundamental counting principal, permutations, combinations, and related probabilities). In this chapter I relate study findings to the research questions I sought to address by means of this study. I then turn to a reiteration of the need for studies such as this one, as evidenced by the literature in the field, while delineating limitations of this study as well as implications of this study for future research.

## Summary of findings

The original research questions and impetus of this study were introduced in Chapter one. These questions were:

1. Do students required to write expository descriptions of procedures involved in solving problems in mathematics, in addition to learning how to solve these problems, show evidence of better understanding of solving the problems in the short-term than do students who solve the math problems with no writing requirement, or students who solve the problems and engage in another form of

writing (novel problem writing), after controlling for differences in initial understanding of the math concepts under consideration?

2. Do students required to write topic-related and task-related novel mathematics problems, in addition to learning how to solve these problems, show evidence of better understanding of solving the problems in the short-term than do students who solve the math problems with no writing requirement, or students who solve the problems and engage in another form of writing (expository writing), after controlling for differences in initial understanding of the math concepts under consideration?

In essence, I was looking to determine if two forms of writing resulted in differential effects on student learning of mathematics concepts. To answer this question I measured the impact of writing on math learning on a convenience sample of 31 college freshman enrolled in a mathematics content course in a School of Education. These 31 students, after completing a pretest to measure initial understanding of the math concepts of interest to this study, completed 3 homework problem sets, after being instructed on the concepts in the classroom, that required them to either engage in expository writing, novel problem writing, or no writing at all. Subjects completed their participation by taking a subtest to measure gains in their understanding of the three math concepts. ANCOVA procedures were used to determine if there were significant differences in the performance of subjects in the three groups.

Addressing the first research question, data analyses indicated that students engaged in expository forms of writing performed no differently statistically than students engaged in novel problem writing or those engaged in no writing. ANCOVA procedures were conducted first to consider performance overall on the posttest measure, and then separately on each of the three categories of problems included on the posttest: standard deviation, fundamental counting principal, and permutations and combinations. Though mean differences were not significant, students in the Expository Writing group, it is noted, earned the highest overall posttest mean scores. In addition, for the analyses with the greatest effect size of those conducted ($\eta^2 = .041$), that which related to solving problems in the permutations and combinations category, those in the Expository Writing group earned the highest mean outcome scores. The ANCOVA analysis indicates that these differences in mean scores was due to chance, though it is also noted that the ANCOVA analyses conducted yielded low power statistics in all cases, indicating that there is a high probability (ranging between 86.4% and 94.2% for the various analyses) that this study lacked the power to detect an effect if it exists in reality. These trends in posttest score means, therefore, may prove useful for consideration in future replication studies with greater power.

Addressing the second research question, ANCOVA procedures lead to the conclusion that there was no significant difference in performance for students engaged in novel problem writing in comparison to those engaged in expository writing or no writing. This result was found when considering overall student performance on the posttest measure, and when performance on each category of math problem was considered. As indicated above, low power statistics indicate that this study may have failed to detect significance where it might actually exist.

**Relating the present study to the field**

In the most general sense, this study addresses the overall need for an increased number of studies in the area of teaching and learning practices in the mathematics classroom. Given the resounding call from government officials and education professionals for educators to use instructional practices that have been substantiated through research, combined with the relatively incomprehensive collection of such practices currently in mathematics education, considerable studies that address the effectiveness of particular mathematics instructional practices across the spectrum are needed.

While reviewing the literature of the field, I also observed that the majority of research studies in the field on writing in mathematics were found to be qualitative in nature centering on measuring the perceived benefits of writing to learn mathematics. Being perceived to make a difference and actually making a difference are not congruent concepts. With this in mind, it appears evidently critical that more comparative studies that include a control group, and address the relative effects of writing on mathematics learning are needed in the field. As indicated earlier, Porter and Masingila (2000) recognized this premise when designing their study on writing in mathematics. My study was designed to address this need. This study addressed the problem of poor student performance in mathematics in that it sought to lend evidence to the question of whether writing in mathematics is a tool that promotes student understanding of mathematical concepts. Though the present study did not produce significant differences in the learning outcomes of students engaged in expository writing, novel problem writing, or no writing at all, the findings of this study do provide insight. It is possible that, for instance, despite findings of perceived benefit in relation to writing in mathematics, certain forms of writing in mathematics may not result in significant improvement in math learning. Indeed, Porter and Masingila (2000) found no significant differences in learning for students engaged in writing along with other course elements in comparison to those who did everything required except the writing. Of course this conclusion has not been substantiated, but the present study points to it as a possibility. In sum, this study contributes to the literature by serving as a comparative study that considers the relative effect of using two forms of writing (or no writing) to learn mathematics concepts. Several writings in the field stress the need for additional studies that can give teachers more direction for using writing in mathematics effectively. This study addresses this need.

**Study limitations**

A number of limitations to this study exist. To begin, this study relied on data collected from a sample of convenience. The students enrolled in the two sections of the college course where this study was conducted, though not identical in nature, do not represent the general population. By virtue of the University's requirements for admission alone, students are necessarily matched in measurable and immeasurable ways In addition, the students chose to enroll in the course because it was of interest to them, or at the very least, it was required for them based on a general interest in a related area of study (i.e. math teaching). A sample that could have been randomly selected from the general population, or even from the general University population, may have been affected by the activities of the writing groups in different ways.

Another limitation to this study was the small size of the sample. Only 31 subjects were included in the data analysis, and these students were divided into three treatment groups. Larger sample sizes could have led to increased power. Increased power would allow one to more confidently conclude that a particular study's results are reliable.

Another limitation of the study is in relation to the age of the participants. The undergraduate students have all completed a regiment of compulsory learning and have had the chance to move towards crystallizing their optimum learning styles. By the time a student reaches post-secondary education they may be self-aware enough to have developed a preferred style of learning, and may be resistant to or confused by styles of learning that they have not chosen as optimal for their own learning. In addition, the undergraduate students were exposed to many if not all of the topics included in this study earlier in their learning. It is possible that, because of this previous exposure to the topics, students interface with the material in a different way than a student who is learning the material for the first time might. In other words, younger students who are introduced to the topic for the first time may engage in the learning process differently, and writing to learn at this juncture may have a different effect. Younger students are also theoretically at less advanced stages of learning, and may be aided (or hindered) by the writing process in ways different from older students who have achieved more advanced stages of learning.

Another limitation of this study was the duration of time during which the forms of writing were being practiced. Subjects engaged in the writing tasks over the span of about two weeks for only three problem sets. Though the students are normally introduced to the topics in this study at approximately the same pace, the fact that they engaged in writing for a relatively short period of time may have limited its effects on their learning. In addition, though the topics included in this study are normally covered as a part of the content of the math course, they are not necessarily covered as early in the semester as they were. The changing of the typical order of instruction may represent a limitation.

This study did not measure any long-term effects of the writing on math learning, which represents another limitation. It is possible that short-term and long-term effects could differ based on the writing group assignment. That is to say, though I found no significant differences in learning between groups in the short-term, in the long-term the effect might have been different.

Another limitation of this study is that it did not allow for the teaching of the various writing styles. The writing styles were explained to students, but they did not have a chance to solidify their understanding of the forms of writing prior to using them within the parameters of the study. Perhaps there existed a learning curve of sorts in relation to using the styles of writing, and some students may not have been confident in what they were doing in relation to the writing tasks. This possible uncertainty may have led to a replacement of learning the math concepts, partially or wholly, with learning the writing styles. There was no study design control to account for differences in familiarity with the various writing styles, nor was there statistical control of this variable in the guise of a covariate.

This study also did not evaluate the work completed on the homework problem sets for correctness or quality, but only for completion. It is possible that those subjects not excluded from the sample for analysis, though they completed all assigned problem sets, did not complete them accurately. Though the design of this study did not allow for correcting all homework sets and providing feedback in relation to how well a subject was completing the required writing task, a study that includes this element might find that the writing impacts learning in different ways from what was revealed in this study.

The observation that the magnitude of performance by the writing groups differed based on the topic being studied introduces another possible limitation. When designing this study a variety of math concepts were selected because they were areas in which students traditionally struggled based on the instructor's observations over the years (25+ years). I set out to measure general math performance overall, but ended by considering performance overall and in each concept area based on observations that differences in performance in concept areas were not consistent across the types of math problems. It may have been prudent, for instance, to measure the effects of writing on one math topic, say permutations and combinations, over the entire period of the study, or to measure the effects of writing on learning topics that are more closely paired.

**Implications for future studies**

A number of implications for future studies are evident. To begin, the conclusion that this study was found to have little power in detecting a treatment effect, if it exists, indicates that findings of this study can not be accepted with great confidence. That is to say, though no significant effect was found in this study, I can not confidently conclude that this effect does not exist in the sample population given the study's low power. This suggests that a replication study, perhaps with a larger sample size, is warranted.

Subjects participating in this study were not chosen randomly from the population. This introduces another implication for future studies. A replication study that allows for the random selection of participants might yield different results. This seems particularly relevant since at the undergraduate level, students, to some degree, select their courses based on interest. In primary and secondary schools, all students, regardless of their interests in mathematics, are required to learn certain concepts in mathematics. Students under this circumstance may represent a potential setting for future studies to take place.

Younger students, in addition, being introduced to many concepts in mathematics for the first time, and being at less advanced stages of learning, may respond differently than the present sample when writing is paired with mathematics learning. This observation implies that future studies that measure the effects of math learning of younger students at various stages of learning might be justified.

As observed earlier, consideration of mean posttest scores, though found in this study to not be statistically different, indicate that subjects assigned to the different writing groups performed inconsistently on math problems grouped by content. This observation introduces another implication for future studies. Though these differences must be attributed to chance in this study, the fact that this study lacked sufficient power leads to the possibility that this study failed to detect actual differences in achievement. It could be, therefore, that writing may affect the learning of different topics in mathematics in different ways. In particular, it appears that a study concentrating on the effects of writing on one main form of math learning may be warranted. This observation also suggests that writing in mathematics might be effective when learning some concepts, but may be ineffective or even prohibitive when learning other concepts. It appears necessary then to conduct additional studies that systematically measure the effects of writing on learning specific topics in mathematics.

Numerous studies exist that consider the perceived benefits of writing to learn in mathematics. Few studies consider the comparative effects of writing on math achievement, and even fewer studies consider both perceived benefits and comparative effects and whether these correspond. Future studies that consider if perceived benefits of writing to learn mathematics correspond to actual gains in learning mathematics appear to be needed.

A further implication of this study for future research can be related to how a student's understanding of the writing practice may affect how the writing assists in or impedes learning. A student unfamiliar or uncomfortable with the various writing styles may be less focused on writing to learn the mathematics and more focused on how to accomplish the writing itself. Additional studies that ensure equal achievement in the ability to write in the mathematics classroom might be useful.

In conclusion, though the findings of this study to measure the impact of writing on mathematics learning failed to detect any significant differences in the performance of students in three different writing groups (Expository Writing, Novel Problem Writing, and No Writing), this study does contribute to the field of mathematics education research in that it addresses the need for increased investigations of teaching and learning in the mathematics classroom. This study provides a comparative study of the effects of writing on math learning, filling the need for an increase in this class of study in relation to classroom-based writing-to-learn practices. In addition to meeting these pertinent needs, this study, despite its limitations, also suggests numerous implications for future studies designed to measure the effects of writing on mathematics learning.

# APPENDIX A

# CONSENT PROCESS SCRIPT

Towards Evidence-Based Practices in Mathematics Instruction: Investigating the Impact of Writing on Student Ability to Solve Mathematics Problems

Consent Process Script

Recruitment of Student Participants: Description of Study and Consent Process

(Introduction)

Investigator: Good morning and thank you for your attention. I am here today to describe to you a study that I am initiating to discover how writing in mathematics might impact student understanding of math concepts. This study will be used to fulfill the requirement of my dissertation.

(Purpose/Goals)

Investigator: You have undoubtedly heard it said that American students are falling behind students in other countries in mathematics achievement. Because of this, as you might imagine, educators have been trying new approaches to teaching mathematics. Also, laws and policies have been changed to require teachers and schools to use techniques that have been shown through research to work. One method that has often been used by math teachers from elementary thorough college is writing. Teachers have introduced writing into their courses in several ways as a means to help students better comprehend that math that they were learning. The purpose of my study is to consider through research if two forms of writing, when combined with traditional forms of instruction, help students to better understand the concepts they are studying.

(Methods)

Investigator: To measure this I am looking for students enrolled in this course to volunteer to participate in this study. If you choose to participate you may be asked to do alternative homework problem sets that involve writing. I wish to now give you an overview of the study so that you can better understand what your role will be if you choose to participate.

Participants in this study will be asked to complete a 7-item pretest, 4 problem sets of 2 to 5 problems each, and a 7-item posttest. The math topics to be included on these items are Standard Deviation, The Fundamental Counting Principle, Permutations, Combinations, and Determining Probabilities. These 4 topics are topics that are normally covered as a part of this course. Your instructor will cover these topics as part of this course's instruction. If you choose to participate in this study, after taking the pretest, which will be administered today following this discussion, you will be assigned to one of three groups. Each group will be given a different problem set to complete following each of 4 lessons. If you choose not to participate in the study, you will complete the problem set assigned by your professor.  All students will be covering the same topics at the same time, the only difference will be the problem sets that you complete. Those participating in the study will be given either a problem set that requires no writing at all (this is the control group), a problem set that asks you to explain how you found your answers in writing (this is the expository writing group), or a problem set that asks you to write new math problems (this is the novel problem writing group). After completing the four problem sets you will be asked to complete the post test. The entire process will be completed within three weeks. In addition, Dr. Schiller will not know which students have chosen to participate, and I will not know the identity of those who participate. All students in this course will be assigned an ID number, whether you participate in the study or not. You will use this ID number on the 4 problem sets completed during the course of the study. This will be true for each student regardless of which problem set you complete, either as part of the study or if completing the problem set assigned by your instructor. Students will turn in their problem sets in an envelope in class only putting their ID numbers on their work. I will retrieve the envelope and report to Dr. Schiller the ID numbers of those completing problem sets, but not which problem set you completed. You will receive credit for homework completion regardless of which problem set you complete. Dr. Schiller will have a list of students and ID numbers and thus will be able to determine who has completed a problem set, but not which problem set they completed.

(Pause)

Investigator: Are there any questions so far? (*Address student questions)*.

(Explaining Consent)

Investigator: Now that I have described the study to you, I would like to describe the consent process and your rights as a willing participant. I have prepared a consent form which describes in detail your role as a volunteer participant and your rights. We will momentarily go over this form, but I wish to point out two of the most important points. First and foremost I wish for you to understand that your participation is completely voluntary and that if you choose not to participate you will not be penalized and your grade will not be affected in any way. I also want you to understand that even if you volunteer to participate, you can choose to stop participating at any time during the course of the study. I will now pass out the consent form and we will go over it together. (*Distribute and read consent form.)*

(Potential Participants Expression of Understanding and Questions)

Investigator: Are there any questions about the consent form, about what giving your consent means, or about your role as a study participant if you choose to participate? (*Address all student questions.)*

Before asking those of you who are willing to participate to sign the informed consent form, I first wish to ask a few questions just to make sure you understand the purpose of the study, what your role will be, and your rights as a volunteer participant:

Please describe to me your understanding of your role in this study if you choose to participate (*allow student discussion/response).*

Please describe for me your options once you agree to participate in the study *(allow student discussion/response).*

Please describe to me your understanding of how participation in this study will impact your grade or your relationship with your professor (*allow student discussion/response).*

.

Investigator: If there are no more questions, I will step out of the room while those of you who wish to participate sign the consent form. You will notice that there are two copies, both signed by me, the investigator. Please sign one copy and place it in this envelope (*indicate envelope)* and keep the other copy for your records. When all those who wish to participate have placed their consent forms in the envelope, will someone please notify me where I will be waiting in the hallway. Thank you for your willingness to consider participation in my study.

(END)

# APPENDIX B

# STUDY PARTICIPANT LETTER OF CONSENT

# CONSENT TO PARTICIPATE IN RESEARCH

**Project Title:** Towards evidence-based practices in mathematics instruction: Investigating the impact of writing on student ability to solve mathematics problems
**Researcher:** *Shaalein C. Lopez*
**Faculty Sponsor:** *Dr. Diane Schiller*

**Introduction:**
You are being asked to take part in a research study being conducted by Shaalein C. Lopez for a dissertation under the supervision of Dr. Diane Schiller in the Department of Curriculum, Instruction, and Educational Psychology of Loyola University of Chicago.

You are being asked to participate because you are currently undergoing training in the area of mathematics. Your growth in the area of mathematics as a result of the training you are undergoing is a potential source of data whose results may contribute to the field of mathematics teaching and learning. You will potentially be one of approximately 40 participants.

Please read this form carefully and ask any questions you may have before deciding whether to participate in the study.

**Purpose:**
The purpose of this study is to determine what effects, if any, two forms of writing have on student learning and achievement in mathematics. The two forms of writing are: (1) writing detailed descriptions of the problem solving process; and (2) writing completely new mathematics problems for others to solve.

**Procedures:**
If you agree to be in the study, you will be asked to:
- Complete a 25-30 minute long seven-item pretest covering 5 mathematics topics.
- Complete 3 modified practice problem sets of 2-5 problems each that may require you to write about mathematics. Each problem set should take from 10-25 minutes to complete. Instruction that you receive in relation to the 5 topics is normally covered as part of the curriculum of your course.
- Complete a 25-30 minute long seven-item posttest covering the 5 mathematics topics.

Consenting participants will be randomly assigned to one of three groups by the course researcher. The problem sets participants are assigned will be determined by group assignment. They are as follows:
- Group 1 will be the control group. Problem sets that this group completes will require math problem solving alone with no additional writing.
- Group 2 will be the expository writing group. Problem sets assigned to this group will require math problem solving combined with the production of written explanations of mathematical procedures used in the problem solving.
- Group 3 will be the novel problem writing group. Problem sets assigned to this group will require math problem solving combined with the writing of related math problems (with solutions) that can be solved by others.

**Risks/Benefits:**
There are no foreseeable risks involved in participating in this research beyond those experienced in everyday life.

There are no direct benefits to you from participation beyond exposure to additional teaching and learning practices that may affect your craft of teaching. In addition, the larger society of mathematics education may benefit by gaining insight into how the writing practices under consideration may affect student mathematical learning.

**Confidentiality:**
At no time will the researcher be aware of your identity in relation to the work you complete as a part of this study. When completing the pretest and posttest you will use a numerical code given to you when you take the pretest. The researcher will receive these instruments, but will have no way of determining who has been assigned which code. Everyone in the class will be assigned a code so that the instructor cannot know who is participating in the study and who is not. Problem sets will be completed using the assigned numerical codes, as will the posttest. The master list of corresponding names and study identification numerical codes will remain in the possession of your instructor throughout the course of study implementation and will be destroyed at the termination of this study.

**Voluntary Participation:**
Participation in this study is voluntary. If you do not want to be in this study, you do not have to participate. Even if you decide to participate, you are free not to answer any question or to withdraw from participation at any time without penalty. If you decide to participate or not to participate, your decision will not favorably or adversely affect your relationship with your course instructor, your grade in the course, the instruction you receive in the course, or your standing at Loyola University in any way. In addition, the researcher implementing the study does not evaluate or grade your performance in anyway that impacts your grade in the course. The work that you do as a part of this study is evaluated without knowledge of your identity, and only within the confines of this study.

**Contacts and Questions:**
If you have questions about this research study, please feel free to contact me, Shaalein C. Lopez, at slopez5@luc.edu, or the faculty sponsor, Diane Schiller, at dschill@luc.edu.

If you have questions about your rights as a research participant, you may contact the Loyola University Office of Research Services at (773) 508-2689.

**Statement of Consent:**
Your signature below indicates that you have read the information provided above, have had an opportunity to ask questions, and agree to participate in this research study**.** You will be given a copy of this form to keep for your records.

______________________________ ______________

**Participant's Signature** **Date**

______________________________ ______________

**Researcher's Signature** **Date**

# APPENDIX C

# STUDY PRETEST AND POSTTEST

**TOWARDS EVIDENCE-BASED PRACTICES IN MATHEMATICS INSTRUCTION:**

**INVESTIGATING THE IMPACT OF WRITING ON STUDENTABILITY TO SOLVE MATHEMATICS PROBLEMS**

# PRETEST

STUDY ID: XXXXXX

DATE:

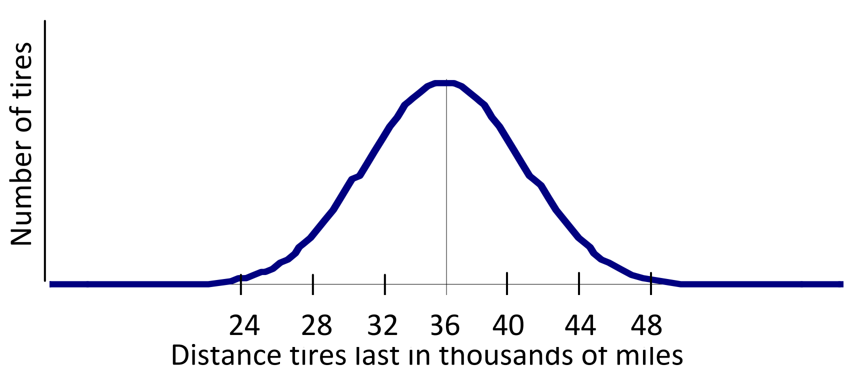
*Thank you for your participation in this study. As indicated previously, the researcher will not be aware of your identity when reviewing your performance on this pretest.*

Please check the boxes that correspond to the mathematics courses you completed in high school:

| High School Course | General Course | Advanced Placement Course |
|---|---|---|
| Algebra 1 | | |
| Geometry | | |
| Algebra 2/ Trigonometry | | |
| Pre-Calculus | | |
| Calculus | | |
| Other Math Course 1: | | |
| Other Math Course 2: | | |
| Other Math Course 3: | | |
| Other Math Course 4: | | |

**Topic 1: Standard Deviation**

1. A company produces car tires that last an average (mean) distance of 36,000 miles. The distances that tires last are normally distributed as represented in the following normal curve:



YOUR ANSWERS:

a. According to the above graph, what appears to be the value of the standard deviation of the distribution?

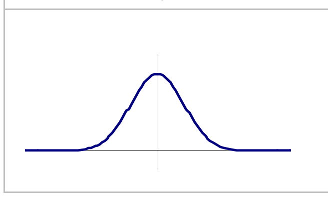
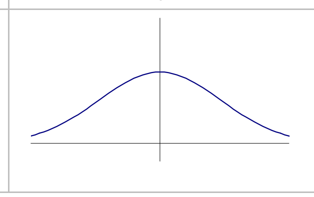
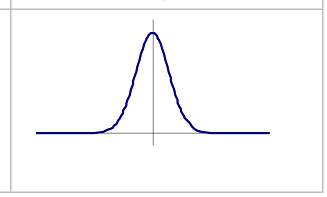
1a. 4,000 (miles)

b. Given that the above distribution is normal, what percentage of tires last between 32,000 and 40,000 miles?

1b. About 68%

*(Adapted from Mathematics: A Human Endeavor; Third Edition, Chapter 9)*

2. Assuming that the scales on all of the below graphs are identical, which normal curve graph appears to have the greater standard deviation?

| a. | b. | c. |
|---|---|---|
|  |  |  |

YOUR ANSWER:

2. b.

3. Following are a list of quiz scores for Classroom A.

| | | | | |
|---|---|---|---|---|
| 7 | 9 | 4 | 5 | 9 |
| 8 | 7 | 5 | 6 | 10 |

YOUR ANSWERS:

a. What is the mean of this set of quiz scores?

3a. 7 (points)

b. What is the standard deviation of this set of quiz scores?

*(Your answer can be left in terms of a square root.)*

3b. $\sqrt{3.6}$ (points)

**Topic 2: Probability and the Fundamental Counting Principal**

4.

YOUR ANSWERS:

a. You wish to purchase a Lunch Counter meal for your lunch. Each meal includes one sandwich, one salad,

4a. 280

and one drink. The Lunch Counter offers 5 types of sandwiches, 7 types of salads, and 8 types of drinks. How many different meals are possible?

b. The 5 types of sandwiches include: ham, turkey, veggie, PB&J, and roast beef. If you ask a friend to pick your meal for you, what is the probability that your meal will contain a ham sandwich?

4b. 1/5 or 20%

**Topic 3: Permutations, Combinations and Probability**

.

5.
Ten films have been entered in the film festival competition. Trophies are given for first, second, and third place. If there are no ties allowed in the competition, in how many different ways can the three trophies be awarded?

YOUR ANSWER:

5. 720

6.
A student entering fourth grade has to read three books over the summer. The teacher lists six books from which the student can choose. How many different sets of three books could the student select?

YOUR ANSWER:

6. 20

7.
A student needs 2 notebooks of different colors for class. The student can choose from the following colors: red, blue, green, orange, and yellow. What is the probability that a student's set of notebooks will include a green one?

YOUR ANSWER:

7. 2/5 or 40%

| | |
|---|---|
| Your Study Identification Number: | XXXXXX |

*Please tear off your study identification number and* retain it for your use throughout the study.

**TOWARDS EVIDENCE-BASED PRACTICES IN MATHEMATICS INSTRUCTION:**

**INVESTIGATING THE IMPACT OF WRITING ON STUDENTABILITY TO SOLVE MATHEMATICS PROBLEMS**

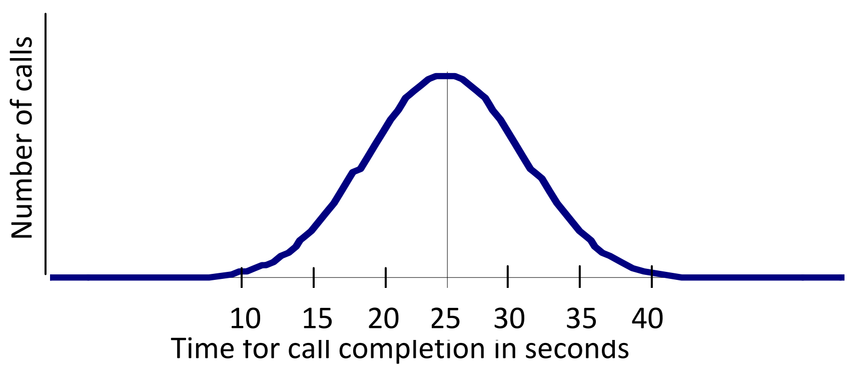
# POSTTEST

| STUDY ID: | DATE: |
|---|---|

*Thank you for your participation in this study. As indicated previously, the researcher will not be aware of your identity when reviewing your performance on this pretest.*

**Topic 1: Standard Deviation**

4. Telephone operators complete directory assistance calls in an average (mean) time of 25 seconds. The amount of time all operators take to complete these calls is represented in the following normal curve:




YOUR ANSWERS:

c. According to the above graph, what appears to be the value of the standard deviation of the distribution?

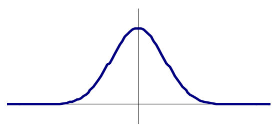
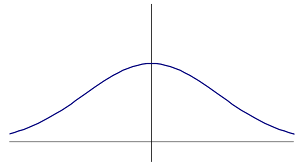
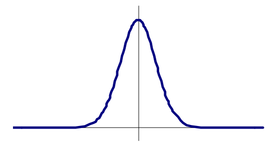
1a. 5 (seconds)

d. Given that the above distribution is normal, what percentage of calls took more than thirty seconds?

1b. About 16%

*(Adapted from Test Bank for Mathematics: A Human Endeavor; Third Edition, Chapter 9)*

5. Assuming that the scales on all of the below graphs are identical, which normal curve graph appears to have the smallest standard deviation?

| a. | b. | c. |
|---|---|---|
|  |  |  |

YOUR ANSWER:

2. c.

6. Following are a list of times in seconds that it took the students in a fifth grade class to complete 15 sit-ups.

| | | | | |
|---|---|---|---|---|
| 16 | 19 | 20 | 18 | 23 |
| 21 | 17 | 22 | 16 | 18 |

YOUR ANSWERS:

c. What is the mean of this set of quiz scores?

3a. 19

d. What is the standard deviation of this set of quiz scores?
*(Your answer can be left in terms of a square root.)*

3b. $\sqrt{5.4}$

**Topic 2: Probability and the Fundamental Counting Principal**

4.

YOUR ANSWERS:

c. The local convenience store sells small, medium, and large cups of coffee. Customers can select dark roast, medium roast, or light roast coffee with or without cream. If you order one cup of coffee, how many different choices do you have?

4a. 18

d. Suppose you send a friend to the store to purchase a cup of coffee for you. If your friend picks your coffee at random, what is the probability that your friend will select a large light roast?

4b. 1/9 or 11%

**Topic 3: Permutations, Combinations and Probability**

.

5.

Ten cooks entered the State Fair Chili Contest. Ribbons are given for first, second, and third place. If there are no ties allowed in the contest, in how many different ways can the three ribbons be awarded?

YOUR ANSWER:

5. 720

6.

For the final examination each student had to submit four culminating assignments. The instructor allowed students to select the 4 assignments from a list of 10 possibilities. How many different sets of four assignments could a student select?

YOUR ANSWER:

6. 210

7.

A student needs 2 notebooks of different folders for class. The student can choose from the following colors: red, blue, green, orange, black, and yellow. What is the probability that a student's set of folders will include a green one?

YOUR ANSWER:

7. 5/15 or 1/3

APPENDIX D

STUDY LESSONS AND PROBLEM SETS

# LESSSON ONE

TOPIC: STANDARD DEVIATION

GOAL: To introduce standard deviation and its fundamental properties and to introduce procedures for calculating the standard deviation of a set of values.

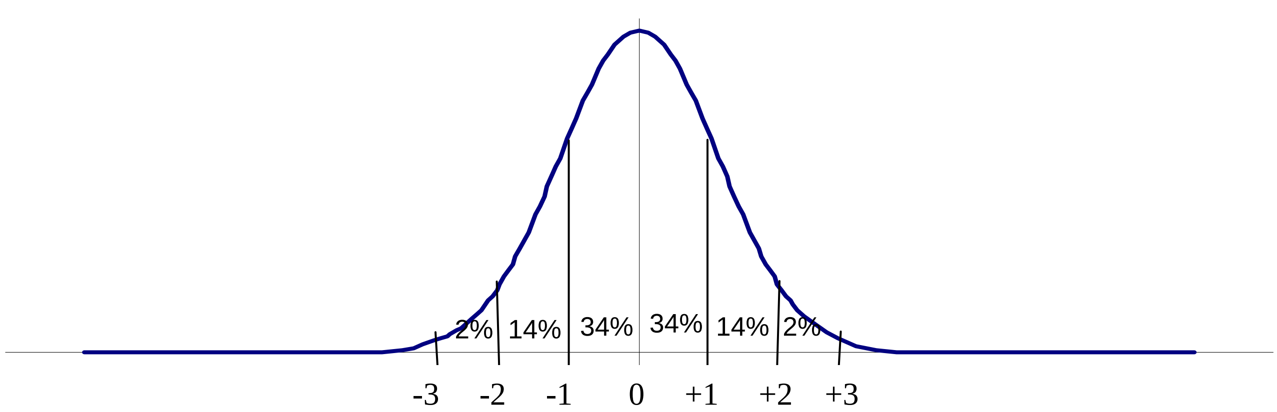
I. Definition of Standard Deviation
II. Standard Deviation and Normal Distribution
III. Determining Standard Deviation

I. Definition of Standard Deviation

Standard Deviation – In statistics, a measure of how much the data in a certain set are scattered around the mean. It is a measure of dispersion of a set of data from its mean.

II. Standard Deviation and Normal Distribution

Normal Distribution – The frequency distribution of many natural phenomena (e.g. population height) represented by a symmetrical bell-shaped curve (normal curve). The shape of the normal curve demonstrates the notion that measures are usually near the average, but occasionally deviate by large amounts. In a normal distribution, about 68% of values in the data set are within one standard deviation of the mean, about 96% of values are within two standard deviations of the mean, and nearly 100% of values are within three standard deviations of the mean. The lower the standard deviation is the closer the values in a data set are clustered.



III. Calculating the standard deviation of a set of data

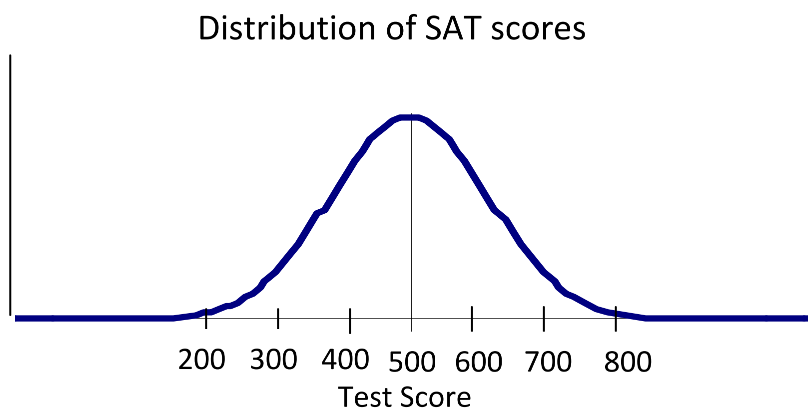
The standard deviation is also defined as the square root of the arithmetic mean of the squares of the deviations from the arithmetic mean. The standard deviation of a set of numeric data is determined by performing the following steps:

1. Determine the mean of the data set.
2. Determine the difference between each value in the data set and the mean.
3. Determine the square of each difference found in step #2.
4. Determine the mean of the squares found in step #3.
5. Determine the square root of the mean found in step #4.

**Lesson 1 Problem Set: Group #1**

1.* When a large set of numbers fits a normal curve, what percentage of the numbers are within one standard deviation of the mean? _about 68%___

2.* The following normal curve represents the normal distribution of student SAT scores.



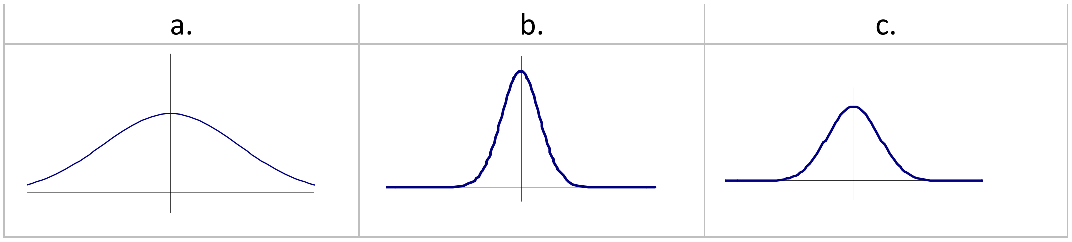

A) What percentage of scores were between 500 and 600? _about 34%___

B) What percentage of scores were between 300 and 600? _about 82%___

C) What percentage of students have scores greater than 600? _about 16%

3. Assuming that the following curves are represented on the same scale, order the curves in increasing order of standard deviation.

_____b_____ _____c_____ _____a_____




4.* Here are the times, in seconds, that it takes 10 runners to run an 800-meter race.

| | | | | |
|---|---|---|---|---|
| 114 | 116 | 119 | 116 | 120 |
| 120 | 121 | 125 | 121 | 128 |

A) Determine the standard deviation of the data set. ____4_____

B) What percentage of scores are two standard deviations or less from the mean? ___about 96%_____

5.* Suppose in a second race two of the times changed so that the fastest runner decreased his time to 104 and the slowest time increased to 138 so that the data set is now:

| | | | | |
|---|---|---|---|---|
| 104 | 116 | 119 | 116 | 120 |
| 120 | 121 | 125 | 121 | 138 |

A) Determine the standard deviation of this new data set. ____8_____

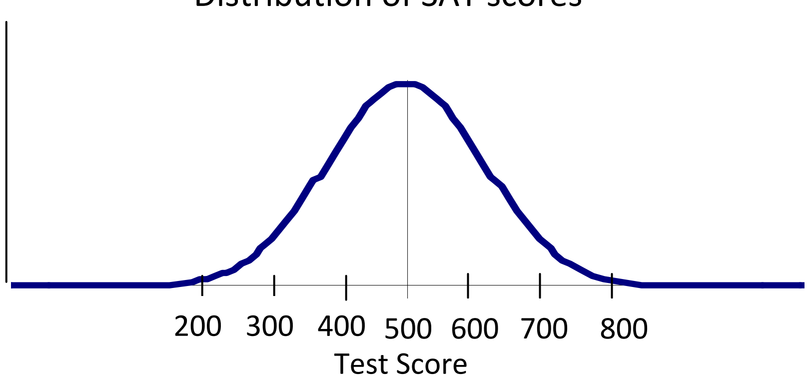
B) What was the difference in value between the standard deviations of the two data sets? ____It doubled_____

**Lesson 1 Problem Set: Group #2**

1.* When a large set of numbers fits a normal curve, what percentage of the numbers are within one standard deviation of the mean? ______________
Explain your answer in writing:

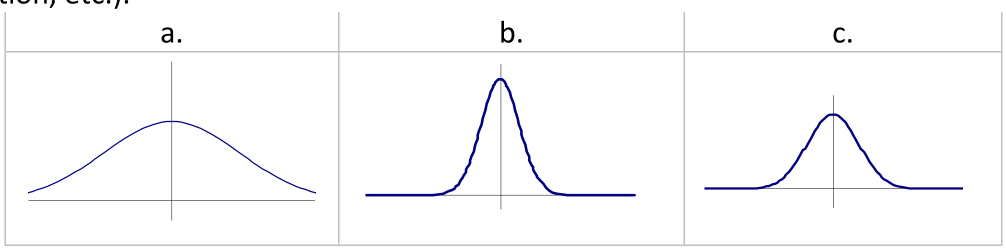
2.* The following normal curve represents the normal distribution of student SAT scores.

Distribution of SAT scores



In the space below determine the percentage of scores between 500 and 600. Also determine the percentage of scores between 300 and 600 as well as the percentage of scores greater than 600. Discuss in writing the steps taken to determine each percentage.

3. Assuming that the following curves are represented on the same scale, discuss the difference in standard deviation among the curves and how this can be determined from characteristics of the curves. Be sure to compare the relative values of the standard deviation represented in each curve (i.e. which curve has the greatest standard deviation, etc.).




4.* Here are the times, in seconds, that it takes 10 runners to run an 800-meter race.

| | | | | |
|---|---|---|---|---|
| 114 | 116 | 119 | 116 | 120 |
| 120 | 121 | 125 | 121 | 128 |

A) Determine the standard deviation of the data set. Describe in writing each step taken to find the standard deviation as you perform them.
B) What percentage of scores are two standard deviations or less from the mean? Describe in writing how you were able to determine this.

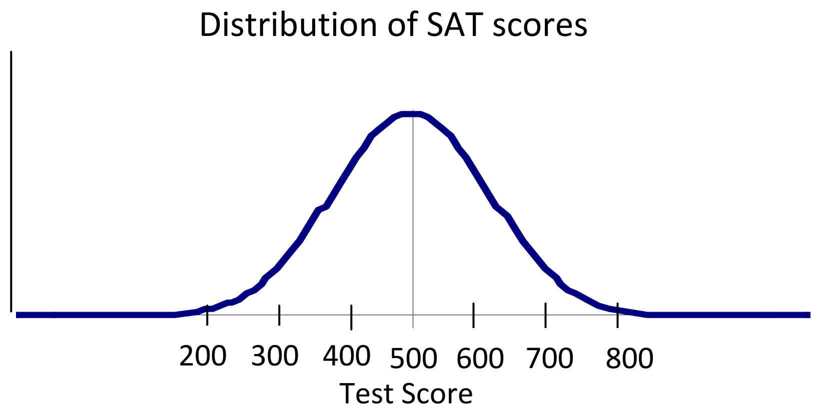
5.* Suppose in a second race two of the times changed so that the fastest runner decreased his time to 104 and the slowest time increased to 138 so that the data set is now:

| | | | | |
|---|---|---|---|---|
| 104 | 116 | 119 | 116 | 120 |
| 120 | 121 | 125 | 121 | 138 |

A) Determine the standard deviation of this new data set. Describe in writing how you found the standard deviation.
B) What was the difference in value between the standard deviations of the two data sets? Explain in writing the difference you found.

**Lesson 1 Problem Set: Group #3**

1.* The following normal curve represents the normal distribution of student SAT scores.




Create three "word problems" in the space below that ask the student to determine the percentage of test scores between two scores on the above graph. Provide the correct answer to your word problem with an explanation.

2.* In the space below create a "word problem" that requires asks the student to determine the relative values of the standard deviations of three normal curves drawn on the same scale. Sketch the curves you wish the student to compare in the boxes provided. Provide the correct answer to your word problem with an explanation.

| a. | b. | c. |
|---|---|---|
| | | |

3.* Here are the times, in seconds, that it takes 10 runners to run an 800-meter race.

114 116 119 116 120

120 121 125 121 128

C) Determine the standard deviation of the data set. __________
D) What percentage of scores are two standard deviations or less from the mean?

4.* Write a “word problem” whereby you make 2 changes to the above data set so that the standard deviation of the new set is double that of the set above. In your word problem you want to present the changes in the data set, ask the student to find the new standard deviation, and ask the student to comment on the difference between the two standard deviations. Provide the solution to your word problem as well as a written explanation of the difference in standard deviation values.

# LESSSON TWO

TOPIC: THE FUNDAMENTAL COUNTING PRINCIPAL AND PERMUATIONS
GOAL: To introduce procedures for using the fundamental counting principal and for determining the permutations of events

I. Using the Fundamental Counting Principal
II. Determining the Number of Permutations of an Event

I. Using the Fundamental Counting Principal

A. Definition: The Fundamental Counting Principal is a basic counting principal that indicates that if there are *a* ways of counting one thing, *b* ways of counting another thing, and *c* ways of counting yet another thing, then there are *a x b x c* ways of doing all three actions all at once.

B. Example: If for lunch there are three types of sandwiches, four types of drinks, and 5 types of salads, then there are 3 x 4 x 5 = 60 types of possible lunches.

II. Determining the Number of Permutations of an Event

A. Definition: A Permutation is the rearrangement of elements of a set in different definite orders. The number of permutations of r things taken from a set of n is

$$_nP_r = \frac{n!}{(n-r)!}$$

B. Example: If six competitors compete for the gold, silver and bronze medals and we wish to know the number of ways the medals can be won, then we wish to know the permutation of six things taken three at a time:

$$_6P_3 = \frac{6!}{(6-3)!} = \frac{6\times5\times4\times3\times2\times1}{3\times2\times1} = 6\times5\times4 = 120$$

**Lesson 2 Problem Set: Group #1**

1.* Colors are produced in computer images by associating each color with a digital code in which each number must be either 0 or 1.

A) If the code consists of 4 digits, how many colors are possible? __16__ $2^4$

B) How many colors are possible if the code consists of 8 digits? __256__ $2^8$

2.* Rowing teams can consist of two, four, or eight rowers.

A) How many different orders can the members of a four-rower team sit in the boat? __24__ $4!$

B) In how many different orders can the members of an eight-rower team sit in the boat? __40,320__ $8!$

C) Does the number of orders rowers can sit in the boat double as the number rowers on the team doubles? __no__

3.* For a little league game there were nine players on a team.

A) In how many ways can the pitcher and catcher be chosen from the nine players?

__72__ $${}_9P_2 = \frac{9!}{(9-2)!} = \frac{9 \times 8 \times 7!}{7!} = 72$$

B) In how many ways can the first, second, and third basemen, as well as the shortstop be chosen from the remaining seven players?__840__

$${}_7P_4 = \frac{7!}{(7-4)!} = \frac{7 \times 6 \times 5 \times 4 \times 3!}{3!} = 840$$

**Lesson 2 Problem Set: Group #2**

1.* Colors are produced in computer images by associating each color with a digital code in which each number must be either 0 or 1.

A) Determine the number of colors possible if the code consists of 4 digits. Explain in writing how you were able to determine your solution.

B) Determine the number of colors possible if the code consists of 8 digits. Explain in writing how you were able to determine your solution.

2.* Rowing teams can consist of two, four, or eight rowers.

A) Determine the number of different orders the members of a four-rower team can sit in the boat. Explain in writing how you found your solution.

B) Determine the number of different orders the members of an eight-rower team can sit in the boat. Explain in writing how you found your solution.

C) Describe in writing what happens to the number of orders rowers can sit in the boat as the number of rowers on the team doubles. Provide a written explanation of the result.

3.* For a little league game there were nine players on a team. A) Determine the number of ways the pitcher and catcher can be chosen from the nine players. Explain in writing how you found your solution. B) Determine the number of ways the first, second, and third basemen, as well as the shortstop can be chosen from the remaining seven players. Describe in writing how you calculated the solution.

**Lesson 2 Problem Set: Group #3**

1.* Colors are produced in computer images by associating each color with a digital code in which each number must be either 0 or 1.

A) If the code consists of 4 digits, how many colors are possible? __16__ $\mathbf{2^4}$

B) Write a "word problem" similar to the one in part A that will require the student to determine the number of colors possible given a code of a length that you specify. Provide the solution with a brief written explanation.

2.* Rowing teams can consist of two, four, or eight rowers.

A) Write a "word problem" asking the student to determine the number of different orders the members of a two, four, or eight-rower team can sit in the boat. B) Write a second "word problem" asking the student to determine the number of orders the members on the team can sit. Write this problem selecting a team size that will enable the student to compare what happens to the number of orders the members can sit if the team size doubles. Ask the student to make this comparison. Provide the solutions to your problems as well as a written explanation of the solutions.

3.* For a little league game there are nine players on a team. The following positions need to be selected: pitcher, catcher, first baseman, second baseman, third baseman, and shortstop. Write two “word problems” that will require the student to determine how many ways these positions can be selected from players on the team (i.e. how many ways can the pitcher and catcher be selected from the nine players). Make sure the student understands that after the first selection is made (i.e. the pitcher and catcher) there are fewer team players from which to choose for the second selection. Provide solutions to your problems as well as a written explanation for the solutions.

# LESSSON THREE

TOPIC: COMBINATIONS; DETERMINING PROBABILITIES USING THE FUNDAMENTAL COUNTING PRINCIPAL, PERMUTATIONS, AND COMBINATIONS

GOAL: To introduce procedures for determining the combinations of events and to use the fundamental counting principal, permutations and combinations to find probabilities

I. Determining the Number of Combinations of an Event
II. Determining Probabilities using The Fundamental Counting Principal, Permutations, and Combinations

I. Determining the Number of Combinations of an Event

A. Definition: A Combination is the arrangement of elements into various groups without regard to their order in the group. A combination is a selection of things in which the order does not matter. The number of combinations of *r* things taken from a set of *n* is

$$ {}_nC_r = \frac{{}_nP_r}{r!} $$

B. Example: If your car radio allows you to program 5 FM stations as your preset stations and there are 15 FM stations to choose from, then to determine the number of different combinations of 5 FM stations you can select you calculate as follows:

$$ {}_nC_r = \frac{{}_nP_r}{r!} = {}_{15}C_5 = \frac{{}_{15}P_5}{5!} = \frac{15\times14\times13\times12\times11}{5\times4\times3\times2\times1} = 3003 $$

II. Determining Probabilities using The Fundamental Counting Principal, Permutations, and Combinations

A. The probability of an event can be determined in the following way:

$$ \textit{probability of an event} = \frac{\text{number of ways in which the event can occur}}{\text{Total number of equally likely outcomes}} $$

B. Example: In a dice game a player wins if they throw a 7 or 11 on the first throw. The probability of winning on the first throw can be found as follows:

$$\frac{number\ of\ ways\ of\ getting\ a\ 7\ or\ 11}{Total\ number\ of\ possible\ rolls} = \frac{4}{6 \times 6} = \frac{1}{9}$$

Lesson 3 Problem Set: Group #1

1.* Ten players try out for the basketball team. Only five players can be selected. How many different sets of five players can be chosen?

_______252_______

$$_{10}C_5 = \frac{_{10}P_5}{5!} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = 252$$

2.* If only 6 players try out for the team, how many different sets of five players can be chosen? _______6_______

$$_{6}C_5 = \frac{_{6}P_5}{5!} = \frac{6 \times 5 \times 4 \times 3 \times 2}{5 \times 4 \times 3 \times 2 \times 1} = 6$$

3.* In poker a flush is a hand of five cards, all of the same suit. If there are thirteen spades, how many different flushes consisting of spades are possible? _______1287_______

$$_{13}C_5 = \frac{_{10}P_5}{5!} = \frac{13 \times 12 \times 11 \times 10 \times 9}{5 \times 4 \times 3 \times 2 \times 1} = 1{,}287$$

4.* A couple has six grandchildren. What is the probability that all six grandchildren are girls? ______1/64________

$$\textit{probability of an event} = \frac{\textbf{number of ways in which the event can occur}}{\textbf{Total number of equally likely outcomes}} = \frac{\mathbf{1}}{\mathbf{2\times2\times2\times2\times2\times2}} = \frac{\mathbf{1}}{\mathbf{64}}$$

**Lesson 3 Problem Set: Group #2**

1.* Determine the number of different sets of five players that can be chosen if ten players try out for the basketball team but only five players can be selected. Explain in writing how you found your solution.

2.* Describe in detail how you would determine the number of different sets of 5 players possible if only 6 players try out for the team. Also indicate the total number of sets possible.

3.* In poker a flush is a hand of five cards, all of the same suit. Determine the number of different flushes consisting of spades possible if there are thirteen spades in a deck. Describe in writing how you derived your solution.

4.* Suppose a couple has six grandchildren. Determine the probability that all six grandchildren are girls. Explain in writing how you found your solution.

**Lesson 3 Problem Set: Group #3**

1.* Determine the number of different sets of five players that can be chosen if ten players try out for the basketball team but only five players can be selected.

2.* Write a “word problem” similar to the one above in which you select the number of players trying out for the team. Be sure to give the solution and a rationale for this solution as a part of your response.

3.* In poker a flush is a hand of five cards, all of the same suit. Write a “word problem” that will require the student to determine the number of different flushes consisting of a specific suit possible if there are thirteen of each suit in a deck. Be sure to give the solution and a rationale for the solution as a part of your response.

4.* Suppose a couple has six grandchildren. Write a “word problem” that will require the student to determine the probability that the grandchildren are of a specific characteristic (i.e. all girls, etc.). Be sure to provide the solution and a rationale as part of your response.

APPENDIX E

RECORD FORM FOR STUDY IDENTIFICATION

NUMBER ASSIGNMENTS

**TOWARDS EVIDENCE-BASED PRACTICES IN MATHEMATICS INSTRUCTION:**

**INVESTIGATING THE IMPACT OF WRITING ON STUDENTABILITY TO SOLVE MATHEMATICS PROBLEMS**

# RECORD FORM
# STUDY IDENTIFICATION NUMBER ASSIGNMENTS

*Instructor: Please allow students in your class to record their ID numbers on this form. Problem Set/Homework completion will be reported to you by the researcher using these ID numbers..*

| | Student ID # | Student Name |
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APPENDIX F

STUDENT EMAIL LIST FORM

**TOWARDS EVIDENCE-BASED PRACTICES IN MATHEMATICS INSTRUCTION:**

**INVESTIGATING THE IMPACT OF WRITING ON STUDENTABILITY TO SOLVE MATHEMATICS PROBLEMS**

# STUDENT EMAIL ADDRESSES

| | Student Name | Student Email Address |
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| 1. | | |
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APPENDIX G

STUDY VARIABLES USED IN DATA ANALYSIS

Study Variables Used in Data Analysis

**TRMNTGRP**

| | | Value |
|---|---|---|
| Standard Attributes | Label | Writing Group Assignment |
| | Type | Numeric |
| Valid Values | 1 | No Writing |
| | 2 | Expository Writing |
| | 3 | Novel Problem Writing |

**PRETOTPRCNT**

| | | Value |
|---|---|---|
| Standard Attributes | Label | Pretest Total Score as Percent |
| | Type | Numeric |
| N | Valid | 31 |
| | Missing | 0 |
| Central Tendency and Dispersion | Mean | 37.7419 |
| | Standard Deviation | 10.23383 |

**PRESDPRCNT**

| | | Value |
|---|---|---|
| Standard Attributes | Label | Pretest Standard Deviation Score as Percent |
| | Type | Numeric |
| N | Valid | 31 |
| | Missing | 0 |
| Central Tendency and Dispersion | Mean | 37.7419 |
| | Standard Deviation | 10.23383 |

**PREFCPPRCNT**

| | | Value |
|---|---|---|
| Standard Attributes | Label | Pretest Fundamental Counting Principle Score as Percent |
| | Type | Numeric |
| N | Valid | 31 |
| | Missing | 0 |
| Central Tendency and Dispersion | Mean | 37.7419 |
| | Standard Deviation | 10.23383 |

**PREPCPRCNT**

| | | Value |
|---|---|---|
| Standard Attributes | Label | Pretest Permutations and Combinations Score as Percent |
| | Type | Numeric |
| N | Valid | 31 |
| | Missing | 0 |
| Central Tendency and Dispersion | Mean | 37.7419 |
| | Standard Deviation | 10.23383 |

**POSTTOTPRCNT**

| | | Value |
|---|---|---|
| Standard Attributes | Label | Posttest Total Score as Percent |
| | Type | Numeric |
| N | Valid | 31 |
| | Missing | 0 |
| Central Tendency and Dispersion | Mean | 37.7419 |
| | Standard Deviation | 10.23383 |

**POSTSDPRCNT**

| | | Value |
|---|---|---|
| Standard Attributes | Label | Posttest Standard Deviation Score as Percent |
| | Type | Numeric |
| N | Valid | 31 |
| | Missing | 0 |
| Central Tendency and Dispersion | Mean | 37.7419 |
| | Standard Deviation | 10.23383 |

**POSTFCPPRCNT**

| | | Value |
|---|---|---|
| Standard Attributes | Label | Posttest Fundamental Counting Principle Score as Percent |
| | Type | Numeric |
| N | Valid | 31 |
| | Missing | 0 |
| Central Tendency and Dispersion | Mean | 37.7419 |
| | Standard Deviation | 10.23383 |

**POSTPCPRCNT**

| | | Value |
|---|---|---|
| Standard Attributes | Label | Posttest Permutations and Combinations Score as Percent |
| | Type | Numeric |
| N | Valid | 31 |
| | Missing | 0 |
| Central Tendency and Dispersion | Mean | 37.7419 |
| | Standard Deviation | 10.23383 |

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# VITA

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